

$$\hat{H}_{\Delta\Sigma} = \begin{pmatrix} E_2 & 0 \\ 0 & E_1 \end{pmatrix} = E_2 \hat{S}_+ \hat{S}_- + E_1 \hat{S}_- \hat{S}_+$$

$$= E_2 |2\rangle\langle 2| + E_1 |1\rangle\langle 1|$$

$$= E_2 \hat{a}_{12}^\dagger \hat{a}_{12} + E_1 \hat{a}_{11}^\dagger \hat{a}_{11}$$

$$= E_2 \hat{c}_2^\dagger \hat{c}_2 + E_1 \hat{c}_1^\dagger \hat{c}_1$$

$$|x\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \quad \langle x| = (\alpha^* \ \beta^*)$$

$$| \downarrow \rangle = |0\rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \langle \downarrow | = \langle 0| = (0 \ 0)$$

$$| \downarrow \rangle = |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \langle \downarrow | = \langle 1| = (0 \ 1)$$

$$| \uparrow \rangle = |2\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \langle \uparrow | = \langle 2| = (1 \ 0)$$

$$\hat{a}_{11} = |1\rangle\langle 1| = \begin{pmatrix} 0 \\ 1 \end{pmatrix} (0 \ 1) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\hat{a}_{11}^\dagger = |1\rangle\langle 1| = \begin{pmatrix} 0 \\ 1 \end{pmatrix} (0 \ 1) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\hat{a}_{22} = |2\rangle\langle 2| = \begin{pmatrix} 1 \\ 0 \end{pmatrix} (1 \ 0) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\hat{a}_{22}^\dagger = |2\rangle\langle 2| = \begin{pmatrix} 1 \\ 0 \end{pmatrix} (1 \ 0) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\hat{a}_{12} = |1\rangle\langle 2| = \begin{pmatrix} 0 \\ 1 \end{pmatrix} (1 \ 0) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \hat{S}_- = \hat{a}_{21}^\dagger$$

$$\hat{a}_{12}^\dagger = |2\rangle\langle 1| = \begin{pmatrix} 1 \\ 0 \end{pmatrix} (0 \ 1) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \hat{S}_+ = \hat{a}_{21}$$

Ποῦ ∃ 1 στον πίνακα;

$$\left[ \begin{array}{cc} \hat{a}_{22} = \hat{a}_{22}^\dagger & \hat{a}_{21} = \hat{a}_{12}^\dagger \\ \hat{a}_{12} = \hat{a}_{21}^\dagger & \hat{a}_{11} = \hat{a}_{11}^\dagger \end{array} \right]$$

$$\hat{a}_{\mu\nu} = |\mu\rangle\langle\nu|$$

$$\hat{a}_{\mu\nu}^\dagger = |\nu\rangle\langle\mu|$$

Στοιχειώδεις διεγέρσεις  
από τη

θεμελιώδη κατάσταση

elementary excitations  
from the ground state

$$\hat{c}_i = \hat{a}_{1i} = |1\rangle\langle i|$$

$$\hat{c}_i^\dagger = \hat{a}_{i1}^\dagger = |i\rangle\langle 1|$$

$$\hat{H}_{T2} = \begin{pmatrix} E_3 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_1 \end{pmatrix} = E_3 |3\rangle\langle 3| + E_2 |2\rangle\langle 2| + E_1 |1\rangle\langle 1|$$

$$= E_3 \hat{a}_{13}^+ \hat{a}_{13} + E_2 \hat{a}_{12}^+ \hat{a}_{12} + E_1 \hat{a}_{11}^+ \hat{a}_{11}$$

$$:= E_3 \hat{c}_3^+ \hat{c}_3 + E_2 \hat{c}_2^+ \hat{c}_2 + E_1 \hat{c}_1^+ \hat{c}_1$$

$$|x\rangle = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \quad \langle x| = (\alpha^* \beta^* \gamma^*)$$

$$|0\rangle = |\emptyset\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \langle 0| = \langle \emptyset| = (0 \ 0 \ 0)$$

$$|1\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad \langle 1| = (0 \ 0 \ 1)$$

$$|2\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \langle 2| = (0 \ 1 \ 0)$$

$$|3\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \langle 3| = (1 \ 0 \ 0)$$

$$\hat{a}_{11} = |1\rangle\langle 1| = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} (0 \ 0 \ 1) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \hat{a}_{11}^+$$

$$\hat{a}_{22} = |2\rangle\langle 2| = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} (0 \ 1 \ 0) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \hat{a}_{22}^+$$

$$\hat{a}_{33} = |3\rangle\langle 3| = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} (1 \ 0 \ 0) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \hat{a}_{33}^+$$

$$\hat{a}_{12} = |1\rangle\langle 2| = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} (0 \ 1 \ 0) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = \hat{a}_{21}^+$$

$$\hat{a}_{12}^+ = |2\rangle\langle 1| = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} (0 \ 0 \ 1) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} = \hat{a}_{21}$$

$$\hat{a}_{13} = |1\rangle\langle 3| = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} (1 \ 0 \ 0) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \hat{a}_{31}^+$$

$$\hat{a}_{13}^+ = |3\rangle\langle 1| = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} (0 \ 0 \ 1) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \hat{a}_{31}$$

$$\hat{a}_{23} = |2\rangle\langle 3| = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} (1 \ 0 \ 0) = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \hat{a}_{32}^\dagger$$

$$\hat{a}_{23}^\dagger = |3\rangle\langle 2| = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} (0 \ 1 \ 0) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \hat{a}_{32}$$

που  $\exists$  1 στον πίνακα;

$$\left[ \begin{array}{ccc} \hat{a}_{33} = \hat{a}_{33}^\dagger & \hat{a}_{32} = \hat{a}_{23}^\dagger & \hat{a}_{31} = \hat{a}_{13}^\dagger \\ \hat{a}_{23} = \hat{a}_{32}^\dagger & \hat{a}_{22} = \hat{a}_{22}^\dagger & \hat{a}_{21} = \hat{a}_{12}^\dagger \\ \hat{a}_{13} = \hat{a}_{31}^\dagger & \hat{a}_{12} = \hat{a}_{21}^\dagger & \hat{a}_{11} = \hat{a}_{11}^\dagger \end{array} \right]$$

$$\hat{H}_{\Delta\Sigma} = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} = E_2 |2\rangle\langle 2| + E_1 |1\rangle\langle 1|$$

$$= E_2 \hat{a}_{12}^{\dagger} \hat{a}_{12} + E_1 \hat{a}_{11}^{\dagger} \hat{a}_{11}$$

$$= E_2 \hat{c}_2^{\dagger} \hat{c}_2 + E_1 \hat{c}_1^{\dagger} \hat{c}_1$$

όπου έδω ως

$$\hat{a}_{\mu\nu} = |\mu\rangle\langle\nu|$$

$$\hat{a}_{\mu\nu}^{\dagger} = |\nu\rangle\langle\mu|$$

$$|1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\hat{a}_{11} = |1\rangle\langle 1| = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \hat{a}_{11}^{\dagger}$$

$$\hat{a}_{22} = |2\rangle\langle 2| = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \hat{a}_{22}^{\dagger}$$

$$\hat{a}_{12} = |1\rangle\langle 2| = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \hat{a}_{21}^{\dagger}$$

$$\hat{a}_{21} = |2\rangle\langle 1| = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \hat{a}_{12}^{\dagger}$$

Στοιχειώδεις Διαχειρίσεις  
από τη  
Θεμελιώδη Κατάσταση

ΕΝΑΛΛΑΚΤΙΚΗ  
ΠΕΡΙΓΡΑΦΗ

$$\hat{c}_i = \hat{a}_{1i} = |1\rangle\langle i|$$

$$\hat{c}_i^{\dagger} = \hat{a}_{i1}^{\dagger} = |i\rangle\langle 1|$$

που  $\exists$  1 στον πίνακα;

$$\begin{bmatrix} \hat{a}_{11} = \hat{a}_{11}^{\dagger} & \hat{a}_{12} = \hat{a}_{21}^{\dagger} \\ \hat{a}_{21} = \hat{a}_{12}^{\dagger} & \hat{a}_{22} = \hat{a}_{22}^{\dagger} \end{bmatrix}$$

$$\hat{H}_{T\Sigma} = \begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix} = E_3 |3\rangle\langle 3| + E_2 |2\rangle\langle 2| + E_1 |1\rangle\langle 1|$$

$$= E_3 \hat{a}_{13}^{\dagger} \hat{a}_{13} + E_2 \hat{a}_{12}^{\dagger} \hat{a}_{12} + E_1 \hat{a}_{11}^{\dagger} \hat{a}_{11}$$

$$= E_3 \hat{c}_3^{\dagger} \hat{c}_3 + E_2 \hat{c}_2^{\dagger} \hat{c}_2 + E_1 \hat{c}_1^{\dagger} \hat{c}_1$$

όπου έδω ως

$$|1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad |2\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad |3\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\hat{a}_{11} = |1\rangle\langle 1| = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \hat{a}_{11}^{\dagger}$$

$$\hat{a}_{22} = |2\rangle\langle 2| = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \hat{a}_{22}^{\dagger}$$

$$\hat{a}_{33}^{\dagger} = |3\rangle\langle 3| = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \hat{a}_{33}^{\dagger}$$

$$\hat{a}_{12}^{\dagger} = |1\rangle\langle 2| = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \hat{a}_{21}^{\dagger}$$

$$\hat{a}_{12}^{\dagger} = |2\rangle\langle 1| = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \hat{a}_{21}^{\dagger}$$

$$\hat{a}_{13}^{\dagger} = |1\rangle\langle 3| = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \hat{a}_{31}^{\dagger}$$

$$\hat{a}_{31}^{\dagger} = |3\rangle\langle 1| = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \hat{a}_{31}^{\dagger}$$

$$\hat{a}_{23}^{\dagger} = |2\rangle\langle 3| = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} = \hat{a}_{32}^{\dagger}$$

$$\hat{a}_{23}^{\dagger} = |3\rangle\langle 2| = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = \hat{a}_{32}^{\dagger}$$

ноū ∃ 1 σov nivaka;

$\hat{a}_{11}^{\dagger} = \hat{a}_{11}^{\dagger}$	$\hat{a}_{12}^{\dagger} = \hat{a}_{21}^{\dagger}$	$\hat{a}_{13}^{\dagger} = \hat{a}_{31}^{\dagger}$
$\hat{a}_{21}^{\dagger} = \hat{a}_{12}^{\dagger}$	$\hat{a}_{22}^{\dagger} = \hat{a}_{22}^{\dagger}$	$\hat{a}_{23}^{\dagger} = \hat{a}_{32}^{\dagger}$
$\hat{a}_{31}^{\dagger} = \hat{a}_{13}^{\dagger}$	$\hat{a}_{32}^{\dagger} = \hat{a}_{23}^{\dagger}$	$\hat{a}_{33}^{\dagger} = \hat{a}_{33}^{\dagger}$