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Inequality in Marx and Piketty
Theory and Policy Implications

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Abstract

This paper explores Piketty's analytical model in contrast to the classical/Marxian approach. The main point raised herein is that Piketty's reliance on neoclassical growth theory is not suitable for the explanation of the dynamics of incomes and wealth. However, certain mainstream economists have attempted to attribute the shortcomings of Piketty's reasoning to some alleged methodological connection with Ricardo and Marx. I argue that Piketty's theory has nothing to do with the classical political economy and Marx. The latter associates endogenous technology and population growth with unemployment and class struggle in determining the wage and profit share. Recent econometric studies indicate that this approach provides a much more rigorous explanation of the dynamics of inequality. Moreover, contrary to the argument of Piketty, the classical approach suggests that labor can regain what it has lost during the neoliberal era through the class struggle.

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I. Introduction

The contributions of A. B Atkinson, Emmanuel Saez and Thomas Piketty in putting together labor and capital income time series going back to the 19th century have gained unanimous recognition in the economic profession. Their work revealed hidden patterns in the evolution of the wage and capital (profit) income shares that removed the last shred of doubt that inequality is a major aspect of contemporary capitalism. In his best-seller *Capital in the 21st Century*, Piketty (2014) presented these data series without the restrictions of space of academic papers together with an attempt of analytical explanation of the patterns appearing therein. Although he relies on neoclassical growth theory, he concluded that inequality is inherent in the system. It results from an increasing capital/income ratio and is aggravated by the gap between the rate of return (profit) on capital (r) and the rate of growth of income (g). Arguing further that the rate of profit will remain roughly constant while the rate of growth will decline, he predicts that inequality will intensify further in the new century. The only way out, in his view, is a wealth tax that will reduce the gap between r and g thereby "democratizing" market economies.

Despite his reliance on neoclassical growth theory, most mainstream economists were hostile to Piketty's conclusions. They considered his assertion that rising inequality is a "law" of capitalism as an attempt to build an argument similar, at least in methodology, to the one in Marx (Acemoglu & Robinson, 2015; Krussel & Smith, 2015). In their view, Piketty's position that the ever-rising importance and concentration of accumulated wealth indicates a source of potential social disorder (Pitketty, 2014, pp. 9-11) implies an inherent contradiction (flaw) for market economies. Moreover, it suggests that modern capitalist societies leave little room for social mobility. People born rich will most likely die richer while the ones born poor will most likely remain poor. This is 180 degrees shift from the earlier views of the profession on inequality. The idea that income convergence is the norm of developed capitalism is abandoned, income polarization is now considered the rule. For neoclassical economists, this is too much.

To understand the last point, we need to look into the history of the academic discussion on inequality. In the golden years of capitalism that followed WWII, mainstream economists argued that inequality would sooner or later belong to the past. In two path-breaking works, Kuznets (1995) and Solow (1956) argued persuasively in this regard. The former suggested that developed market economies will reduce income gaps between households, while the latter claimed that, on top of this, the national economies in the underdeveloped and developing world will converge with those of the developed world. Solow's estimate was never supported by economic data. By 1970 it was more than clear that per capita incomes in developing countries were not converging with those in the developed world and this irrespective of exceptions like Japan or Korea. The evidence was so overwhelming that top mainstream theorists (Lucas, 1988; Mankiw, Romer, & Weil, 1992) admitted the fact and proposed major revisions to neoclassical growth theory. The prediction of Kuznets was not disproven right away, income shares remained relatively stable until the early 1980s. However, the domination of neoliberalism changed things dramatically. Income inequality returned and has intensified since 1980 reaching nowadays the levels that prevailed at the eve of the 20th century. These overwhelming developments place great importance on the competing analytical explanations of inequality and their political and policy implications.

In this context, the present paper explores Piketty's analytical model in contrast to the classical/ Marxian approach. The main point raised herein is that Piketty's reliance on neoclassical growth theory is not suitable for the explanation of the dynamics of incomes and wealth. To support this claim, in second section, I present the neoclassical exogenous growth model which underlies Piketty's analytical reasoning and policy suggestions. By revealing the distinctive assumptions of this model, I discuss the main criticisms he has received from neoclassical economists in third section. In fourth section, contrasting the assertions of mainstream theorists, I argue that Piketty's theory has nothing to do with classical political economy and Marx. The latter associates endogenous technology and population growth with unemployment and class struggle in determining the wage and profit share. Recent econometric studies (Shaikh, 2016) indicate that this approach provides a much more rigorous explanation of the dynamics of inequality. Moreover, contrary to the argument of Piketty, the classical approach suggests that labor can regain what it has lost during the neoliberal era through class struggle. This last point is elaborated in fifth section which summarizes the paper findings.

II. Piketty and the Neoclassical Tradition

Piketty revealed the data patterns that disproved the main analytical conclusions of the post-war mainstream theory (Piketty & Saez, 2003; Piketty, 2014). However, as stated already and elaborated bellow, his theory is not that much different. To understand this, we need to look closer to the original mainstream arguments (Kuznets, 1995; Solow, 1956; 1957) before turning to Piketty's theory.

The paper by Kuznets was, in a similar fashion to Piketty, a presentation of data supporting different conclusions from those prevailing in his time. It indicated that incomes were converging in the developed capitalist world in the first half of the 20^{th} century. This was considered by the author as a rejection of the conclusions of the classical political economy and especially Marx. His theoretical reasoning for the observed convergence of incomes argued that it was the long-term result of capitalist development. He suggested that in the initial phase of capitalist development the migration of peasants to urban areas and the formation of the working class together with the sharp growth of population, due to declining death rates, resulted in accelerated economic growth but also inequality. The latter was based on and intensified by the accumulation of savings at an increasing rate in the hands of the highest-income bracket of the population. However, as capitalist development proceeds the migration of peasants to the cities is reduced, population growth is lessened due to lower birth rates and the rate of savings either remains constant or declines. These changes in the factors controlling inequality lead to the convergence of incomes in developed capitalism.

When it comes to inequality between developed and underdeveloped countries Kuznets is skeptical. Although he stresses that the higher income inequality experienced by underdeveloped countries supports his argument that capitalist development goes together with convergence in incomes, he is not certain that the underdeveloped countries of his day can catch up with the developed world. He argues that due to lower income per capita, compared to the developed world, and the weaker growth rate of income per capita, savings have concentrated in an exceedingly small minority of the population. Consequently, he argues that inequality has increased in the underdeveloped world in the post-colonial era. Moreover, he doubts whether these countries have the appropriate political and institutional structure that will enable them to follow the path of the Northern European countries and the United States.

However, in the mid-1950s mainstream theory did not focus on the impact of institutional and technological factors. The main variables of economic growth and income distribution identified by Kuznets, namely population growth and the rate of savings, were elaborated in the context of a formal mathematical model (Solow, 1956). Solow did not intend to explain income distribution, he aimed to prove that the "warranted" and "natural" rates of growth coincide. It was a reply to the works of Harrod (1939) and Domar (1946)[1]. Nevertheless, the elaboration included important analytical conclusions on economic development and income inequality resulting from the properties of the neoclassical production function.

For the sake of completeness, I will repeat some basic features of the model that are relevant to our discussion. The simplest form involves a closed economy with a Cobb-Douglas production function, constant returns to scale, exogenous population growth, a constant (net) savings rate, and no technical change. Under these assumptions, the model concludes that the rate of growth of output is equal to the rate of growth of the population. Derived from a Cobb-Douglas function, $Y = K^a \cdot (L_0 \cdot e^{n \cdot t})^b$, where $\frac{dK}{dt} = s \cdot K^a \cdot (L_0 \cdot e^{n \cdot t})^b$, $a + b = 1$, and $g = n$, the following two relations hold:

$$\frac{Y}{L} = \left(\frac{s}{g}\right)^{\frac{a}{b}} = \left(\frac{K}{L}\right)^{a} \tag{1}$$

and

$$\beta = \frac{K}{Y} = \frac{s}{g} \tag{2}$$

The notation is standard, Y stands for income which is equal to the value of output, K and L stands for capital and labor respectively, L_0 is the population at time zero, s is the constant rate of (net) savings and α, b are the coefficients of the Cobb-Douglas function. With neutral technical change that is with rising productivity at a constant capital/labor ratio (K/L), the results remain the same with the difference that the steady-state rate of growth is enhanced by the productivity growth rate on top of population growth. Following equation (1), if population growth is reduced, as anticipated by Kuznets, then average income per capita (Y/L) will rise especially if the rate of savings (s) remains relatively constant. Let us now turn to the distribution of income between capital and labor.

[1] In the Harrod-Domar models the "warranted rate of growth" (the rate of growth at which all parties are satisfied with what they have produced) does not converge to the "natural rate", i.e., the sum of productivity and population growth. Solow (1956) solved the problem using the properties of the neoclassical production function. This is the reason his solution triggered what became known as the "capital controversy" (Harcourt, 1972; Felipe & McCombie, 2013). However, there is a second feature of the Harrod-Domar models, the instability of "warranted growth path". Solow never actually took up this matter since he assumed that aggregate supply and demand are always equal by assumption.

We know that due to constant returns to scale ($a + b = 1$) the Cobb-Douglas production function is homogeneous (of degree 1) and the Euler theorem applies. Under these conditions, it can be easily proved that the labor share of Y is equal to b and the capital (profit share) of Y is equal to α. Accordingly, the Cobb-Douglas function, $Y = K^a \cdot (L_0 \cdot e^{n \cdot t})^b$, can be written as follows:

$$Y(t) = w(t) \cdot L(t) + r(t) \cdot K(t) \qquad (*)$$

It bears mentioning that, in equilibrium, the wage is the marginal product of labor ($w = \frac{\vartheta Y}{\vartheta L} = b \cdot \frac{Y}{L}$). Hence, by substituting the original equation and remembering that $b = 1 - \alpha$, we get:

$$Y(t) = (1 - \alpha) \cdot Y(t) + r(t) \cdot K(t) \qquad (*)$$

whereas, by dividing both sides with Y and making use of equation (2), we find:

$$1 = (1 - \alpha) + r(t) \cdot \frac{K(t)}{Y(t)} \qquad (3)$$

which in steady state gives:

$$a = r \cdot \frac{K}{Y} = r \cdot \frac{s}{g} \qquad (4)$$

Equation (4) confirms that the share of capital income is equal to (α), while equation (3) shows that the labor share ($w(t) \cdot L(t)/Y(t)$) is equal to $1 - \alpha = b$. This means that both labor and capital shares remain constant. Taken together with equation (1), where per capita income (Y/L) is expected to increase if population growth declines, this result indicates that everyone will be better off as capitalist development proceeds. Equation (1) indicates also that lower growth rates, besides a higher capital/income ratio, are also associated with a higher capital/labor ratio. Therefore, if Kuznets was right on population growth and the rate of savings, the world would be moving towards higher wages (w) and lower returns of capital (r), as postulated by the law of diminishing returns[2]. If we add to this the empirical findings of the day indicating that the income shares were constant and the wage share was about 80% (Piketty

[2] This is a key neoclassical assumption which concludes that a higher capital-labour ratio (K/L) is followed by a lower return on capital.

2014, p. 227) of the GDP in the developed world, Solow's analytical reasoning was considered supportive to the argument on income distribution presented by Kuznets.

Nevertheless, the Solow model reaches a second conclusion that differs from the predictions of the Kuznets (1955) paper. It argues that growth will bring equality in per capita incomes between underdeveloped and developed countries. In other words, underdeveloped countries will catch up with the developed world. The idea is that if technology is neutral and freely transferable then all countries will eventually share the same capital/labor ratio. It is an extension of the "comparative advantage" theories of international trade where free trade makes all countries equally competitive. In our case, the key factors are the properties of neoclassical equilibrium and the free transfer of neutral technology. The consequent equalization of the capital/labor ratio between countries will bring the equalization of wages and profit rates as in the Stolper-Samuelson theorem (Stopler & Samuelson, 1941). This means that the parameters which determine per capita income, namely the share of labor (b), the share of capital (α), the rate of savings (s) and the steady-state rate of growth (g), will also tend to become equal. In that case, as indicated by equation (1), the per capita income in different countries will tend to become equalized irrespective of any differences in the "initial endowment" of each country[3]. The justification relies again on the law of diminishing returns.

The above is the outline of the optimistic conclusion of neoclassical theory in the "golden fifties". Balanced growth and income convergence were the anticipated dynamics of global capitalism. As far as per capita income convergence is concerned, by 1980 even neoclassical economists were convinced that the dynamics of capitalism were moving in a different direction. This was expressly admitted by Robert Lucas who stated that he was considering *"... the prospects for constructing a neoclassical theory of growth and international trade that is consistent with some of the main features of economic development"* (Lucas, 1988). His paper initiated a line of thought in neoclassical growth theory which abandoned the Solow model in favor of "endogenous growth technology models". The latter are models where the "warranted" and the "natural" growth rates do not necessarily coincide leading to a steady-state growth path. A different stream of neoclassical thought focused on a modification of the Solow model on the grounds that it could explain "conditional convergence" (Mankiw, Romer & Weil, 1992).

[3] In Marxist terminology this means that mainstream economists like Solow believed that the primitive accumulation of capital plays no part in the economic development of a country in capitalism.

Conditional convergence means that the parameters α, b, s, and steady-state g do not become equal between countries, however, technology flows freely, and as result countries converge to different steady states. Nowadays, very few people still stick to the original Solow model. Neoclassical economists mainly work with the "endogenous technology" and "conditional convergence" models on matters of growth. Surprisingly, or not, Thomas Piketty remains one of the few exceptions.

This becomes evident from his views on per capita income dynamics. Piketty, in line with the idea of "absolute convergence", expressly declares that the initial endowment plays no part since "… the poor catch up with the rich to the extent that they achieve the same level of technological know-how, skill, and education…" (Piketty, 2014, p. 71). Moreover, technology flows almost freely since *"...it is often hastened by international openness and trade…"* (Piketty, 2014, p. 71). The only (possible) impediment in knowledge diffusion is the *"…ability of the country to mobilize financing…"* (Source needed). This is a position he holds since 1994. Only inefficient credit markets can stop economic convergence. In his own words: *"No matter how wealthy a family or country is initially; all equally enterprising units of labor would then be able to make the same investments thanks to the credit market. Hence the inequality of initial endowments would not persist"* (Piketty, 2015, p. 57).

However, when it comes to income distribution between capital and labor, he departs from the Cobb-Douglas version of the Solow model but keeps its' other features. Bellow, we will discuss whether Piketty's explanation of the capital/labor income and wealth distribution is consistent with his views on per capita income dynamics. For now, it is worth noting, that Piketty did not hold the same position on the dynamics of capital and labor shares throughout his career. In the introduction for the recent English edition of his 1994 book (Piketty 2015), Piketty admits that he thought that the shares of capital and labor income remained relatively constant. His concerns on inequality were limited to the concentration of property and wealth in a few hands. However, following the millennium he acknowledged that there were variations in the wage and profit shares (Piketty 2001; Piketty & Saez 2013; Piketty, 2014), as well as increasing wage differentials and divergence between capital incomes (Piketty & Saez, 2003). More lengthy time series for France beginning from 1900 indicate that the capital share increased from 1880 to 1930 (it averaged to around 33%), declined from 1930 to 1980 (it averaged around 20% but was only 15% in 1980) and increased from 1980 to 2010 averaging around 27% (Piketty, 2014,

p. 227)[4]. Similar patterns were observed in most countries. In short, the time series of the incomes resemble a U rather than an inverse U predicted by Kuznets. Moreover, the time series of income shares disprove the conclusion of the Cobb-Douglas version of the Solow model, since they are not constant.

As already mentioned above, Piketty does not abandon the analytical concepts and tools of the early neoclassical growth theory (Solow, 1956; 1957) in order to explain this pattern. He considers Solows' equation (4) ($a = r \cdot \frac{K}{Y}$), and equation (2) ($\beta = \frac{K}{Y} = \frac{s}{g}$), respectively, as the first and second "fundamental equations of capitalism". Then he presents time series showing that the capital/income ratio (β) has followed a similar U shaped pattern like the profit share (α) with the difference that the time path of (α) is more "shallow" (Piketty, 2014, p. 165, p. 196, p. 200). Using equation (2) he argues that (β) increases following 1950 because the rate of savings (s) has remained constant while the rate of growth has declined due to a slowdown in population growth and a relatively stagnant exogenous neutral technical change. In this connection, the shallower U of (α), compared to (β), is attributed to a decline in the rate of return on capital (r) under equation (4). In short, Piketty claims that (β) increases faster than any decline in (r) and as a result (due to equation (4)) the profit share (α) increases, although more slowly than the increase in (β).

The smaller decline in (r) is claimed to be due to a greater than unity (1) elasticity of substitution between capital and labor. Note that if the elasticity of substitution is equal to 1, as in the Cobb-Douglas case, then every increase in the capital/income ratio (β) will be followed by an equal decline in the rate of return (r) leaving the capital share (α) the same (Piketty, 2014, pp. 216-217). Piketty argues that the anticipated elasticity of substitution between capital and labor in the 21st century is/will be between 1.3 and 1.6 (Piketty, 2014, p. 221). However, he provides us with no empirical calculation of this measure neither in the text nor in the online appendix (Piketty, 2018, pp. 36-39) not even in his 2014 paper with Gabriel Zucman (Piketty & Zuckman, 2014) that he recommends for further reading. This is an important omission since Piketty does not consider the price of the elasticity of substitution as the result of some law or some other important feature of contemporary capitalism but merely as an empirical finding. It is worth pointing out that Acemoglu and Robinson (2015) report that from a series of recent

[4] It is worth noting that Piketty (2014, p. 220) considers the presentation of long time series data for the capital labour split starting at the late 19th century the main novelty of his study.

empirical studies only one (Karabarbounis & Brend, 2014) finds a capital/labor elasticity of substitution greater than unity.

Moreover, Piketty's analytical elaboration of this issue is also problematic. To explain the relation between β, r and the elasticity of substitution (σ), he uses a constant elasticity of substitution production function (hereafter CES) (Arrow, Chenery, Minhas & Solow, 1961; Piketty, 2014, p.221; 2018, pp. 36-37). The CES is a broader mathematical formulation where the Cobb-Douglas function is a special case. This feature, as well as Piketty's analytical argument, will become evident from the elaboration on the general form of the CES production function, as follows:

$$Y = A \cdot [\delta \cdot K^{-\rho} + (1-\delta) \cdot L^{-\rho}]^{-\frac{1}{\rho}} \tag{5}$$

where, Y, K, and L have the same meaning as above, A is the neutral technology efficiency parameter which we will assume equal to 1 for simplicity, δ is the distribution parameter ($0 < \delta < 1$), and ρ is the substitution parameter. In accordance to Arrow et. al. (1961), the elasticity of substitution of this function is

$$\sigma = \frac{1}{1+\rho} \to \rho = \frac{1-\sigma}{\sigma} \tag{6}$$

by inserting equation (6) into equation (5), we get:

$$Y = \left[\delta \cdot K^{-\frac{1-\sigma}{\sigma}} + (1-\delta) \cdot L^{-\frac{1-\sigma}{\sigma}}\right]^{-\frac{\sigma}{1-\sigma}} \tag{7}$$

by taking the partial derivative of equation (7) with respect to K, we find the rate of return:

$$\frac{\vartheta Y}{\vartheta K} = r = \delta \cdot \left(\frac{Y}{K}\right)^{\frac{1}{\sigma}} = \delta \cdot \beta^{-\frac{1}{\sigma}} \tag{8}$$

By substituting equation (8) into equation (4), we get:

$$\alpha = \delta \cdot \beta^{-\frac{1}{\sigma}} \cdot \beta = \delta \cdot \beta^{\frac{\sigma-1}{\sigma}} \tag{9}$$

Equation (9) summarizes Piketty's argument so far. If the elasticity of substitution is greater than unity any increase in the capital/income ratio (β) will bring an increase in the profit share. However, the increase is smaller than the increase in (β) which initiated it because of $0 < \delta$, $(\sigma - 1)/\sigma < 1$. If the elasticity is equal to 1 then we have the Cobb-Douglas case where the profit share is equal to the distribution parameter δ.

At that point, Piketty forgets the whole theory that underlies this result and treats (β) not as an equilibrium parameter but as a variable and a highly volatile variable as a matter of fact (Piketty 2018, p. 38). He seems to overlook that his definition of $(\beta = s/g)$ and its' relation to (K/L) holds for the case that supply is equal to demand. Therefore, if these relations are always supposed to hold, then aggregate supply must always equal to aggregate demand as well. A heroic assumption to say the least. In case the different values of (β) are associated with different steady-state equilibria then he must explain its' high short-term volatility. The growth rate, in the version of the neoclassical theory he has endorsed, depends on the exogenous rate of growth of population and exogenous technical change. Therefore, it is expected to change very slowly in the short-run and more importantly, its volatility is expected to be limited. In short, the underlying theory does not comply with a constantly changing and highly volatile β.

From the above, it begins to become clear that the analytical scheme Piketty has chosen in order to explain inequality does not fit the purpose. Moreover, his calculations of the capital/income ratio and the rate of profit are highly questionable due to his definition of capital and profit[5]. Nevertheless, the definition of capital is the point connecting the previous discussion and what follows on wealth inequality. In Piketty, capital and wealth are equal in a closed economy with no public wealth (Piketty & Zuckman, 2014). In other words, the parameter $\beta = K/Y = s/g$ is equal or almost equal (in the case of an open economy where people hold foreign assets as well) to the wealth-income ratio (W/Y). Therefore, a higher β implies also a higher wealth/income ratio which leads to a higher concentration of wealth (Morgan, 2015) as I will show shortly. This is the main reason for the "interchangeability" in the use of the terms "capital" and "wealth" in Piketty (2015, p. 47). I will elaborate further on these points at the end of this section since they involve important analytical shortcomings.

[5] Piketty considers capital any form of wealth irrespective of whether it is used in production for profit. His definition does not fit neither the Marxist nor the neoclassical definition. Similarly, when he calculates profit he adds all forms of revenue from capital under his own definition.

Returning to our presentation, Piketty moves on to establish what he calls "the third law of capitalism" by assuming that the steady-state growth trend is efficient. In the Solow world, this means that the economy does not save so much as to push the capital stock to exceed the "golden rule" rate (Phelps, 1961; Mankiw, 2015). This means that the relation $r \geq g$ is expected to hold. However, Solow growth together with income path efficiency implies a representative firm representative agent model that does not tell anything about inequality (Piketty, 2015; Mankiw, 2015). Nevertheless, Piketty sidesteps the problem by suggesting that if idiosyncratic i.i.d. shocks in consumption (savings) are added to the usual set of equations (of a representative agent dynastic model) this can explain high wealth inequality at the steady-state. He argues further that these shock effects are amplified from the difference $r - g > 0$. I will present bellow the Piketty and Zucman (2014b) model in order to elaborate on these matters.

Their model departs from the following **assumptions**:

I. Time is descrete, each period representing a generation (H).
II. Populations are stationary of size 1 $N[0,1]$ and each individual $i \in [0,1]$.

Therefore agregare = average wealth ($W_t = j_t$), and agregare = average income $Y_t = y_t$.

III. Effective labor supply (L_t) grows at an exogenous productivity growth rate (g), i.e., $L_t = L_0 \cdot (1+g)^t$.
IV. Labor income and the rate of return are equal among individuals, $y_{Lti} = y_{Lt,}\ r_{ti} = r_t$.
V. Income=output (closed economy) and are given by a production function $Y_t = F(K_t, L_t)$.
VI. The saving tastes parameter (out of income and wealth) s_{it} is subject to i.i.d. shocks so that $E(s_{ti}) = s' < 1$.

The background **motivation** of their model is following:

Individuals maximize utility choosing between consumption (c_{it}) and wealth (j_{it}). Relatedly, their utility function is:

$$V(c_{it}, j_{it}) = c_{it'}{}^{1-s'_{ti}} \cdot j_{it}{}^{s'_{ti}} \tag{10}$$

Subject to the budget constraint:

$$c_{it} + j_{it+1} \leq y_{Lt} + (1 + r_t) \cdot j_{it} \quad (11)$$

This brings about the following **solution** where utility is maximized for $c_{it} = (1 - s_{ti}) \cdot (y_{Lt} + (1 + r_t) \cdot j_{it})$. By substituting the latter in equation (11), we get:

$$j_{it+1} = s'_{ti} \cdot [y_{Lt} + (1 + r_t) \cdot j_{it}] \quad (12)$$

Given that $y_t = y_{Lt} + r_t \cdot j_t$, by definition, equation (12) can be written as follows on the aggregate:

$$j_{t+1} = s' \cdot [y_{Lt} + (1 + r_t) \cdot j_t] = s' \cdot [y_t + j_t] \quad (13)$$

by dividing equation (13) with $y_{t+1} = (1 + g) \cdot y_t$, and by remembering that $\beta = \frac{K}{Y} = \frac{W}{Y}$ and $a = r \cdot \beta$, we get:

$$\beta_{t+1} = s' \cdot \frac{1-\alpha}{1+g} + s' \cdot \frac{1+r_t}{1+g} \cdot \beta_t = \frac{s'}{1+g} \cdot (1 + \beta_t) \quad (14)$$

In a closed economy, equation (14) always converges to a finite value since the equilibrium rate of return equals the marginal product of capital. The latter declines as β increases like in equation (9) above. The (steady-state) equilibrium value is equal to:

$$\beta^e = s'/(1 + g - s') \quad (15)$$

Piketty and Zucman (2014) attempt to arrive at the familiar result ($\beta^e = s/g$) by suggesting that (s') is the rate of savings out of income Y plus wealth W (see footnote 6) whereas (s) is the rate of savings as a percentage of income alone. However, their algebra is problematic. For

$\beta^e = \frac{s'}{1+g-s'} = s/g$ the following relation must hold: $s = s' \cdot (1+\beta) - \beta$. However, using simple algebra we find that $s = s' \cdot (1+\beta)$[6].

The inconsistency becomes apparent if we look to the possible values of β^e for different values of s'. From equation (15) it is not difficult to ascertain that for normal values of s' (i.e., $s' < \frac{1}{2} \cdot (1+g)$) $\beta^e$is less than unity ($\beta^e < 1$). Keeping in mind that under Piketty's definition in a closed economy $\beta = \frac{K}{Y} = \frac{W}{Y}$, $\beta^e < 1$ means that accumulated wealth (W) is less than annual income (Y). In other words, the model moves between two highly unrealistic assumptions, either savings must be unreasonably high for β to take plausible values, or if savings take normal values then steady-state β is less than 1.

Finally, equation (14) confirms that the capital (wealth) income ratio takes values close to the ratio of a constant rate of savings (s, s') to exogenous growth (g) only in equilibrium. This means that the model does not explore the dynamics of inequality. It is a Solow framework that can lead to a steady-state where the capital income share (α) is high and wealth is very unevenly distributed. The latter will become evident bellow where I conclude the presentation of the model by Piketty and Zucman (2014b).

By departing from the wealth inquality at the steady state, where $z_{ti} = \frac{j_{ti}}{j_t}$ represents the ratio of individual-to-average wealth, dividing equation (12) with $j_{t+1} \approx (1+g) \cdot j_t$, and considertin $\omega = s' \cdot \frac{1+r^e}{1+g}$, we get:

$$z_{(t+1)i} = \frac{s'_{ti}}{s'} \cdot [(1-\omega) + \omega \cdot z_{ti}] \tag{15}$$

The population includes **consumption lovers** (hereafter *cl*), where $s'_{ti} = 0$ with proba $(1-p)$, and **wealth lovers** (hereafter *wl*), where $s'_{ti} = s^* > 0$ with proba p and $s = p \cdot s^*$. The latter begets following equation set:

$$z_0 = 0 \tag{16.1}$$

[6] $s = \frac{S}{Y}$ and $s' = \frac{S}{W+Y}$ by definition and S is total savings. It follows that $\frac{s}{s'} = \frac{Y+W}{Y} = 1 + \beta$ and in a closed economy it follows that $s = s' \cdot (1+\beta)$.

for (cl) or children of (cl), with proba $(1-\mathrm{p})$.

$$\mathrm{z}_1 = \frac{1-\omega}{\mathrm{p}} \tag{16.2}$$

for children of (wl) parents and (cl) grandparents, with proba $(1-\mathrm{p}) \cdot \mathrm{p}$.

$$\mathrm{z}_2 = \frac{1-\omega}{\mathrm{p}} + \frac{\omega}{\mathrm{p}} \cdot \mathrm{z}_1 \tag{16.3}$$

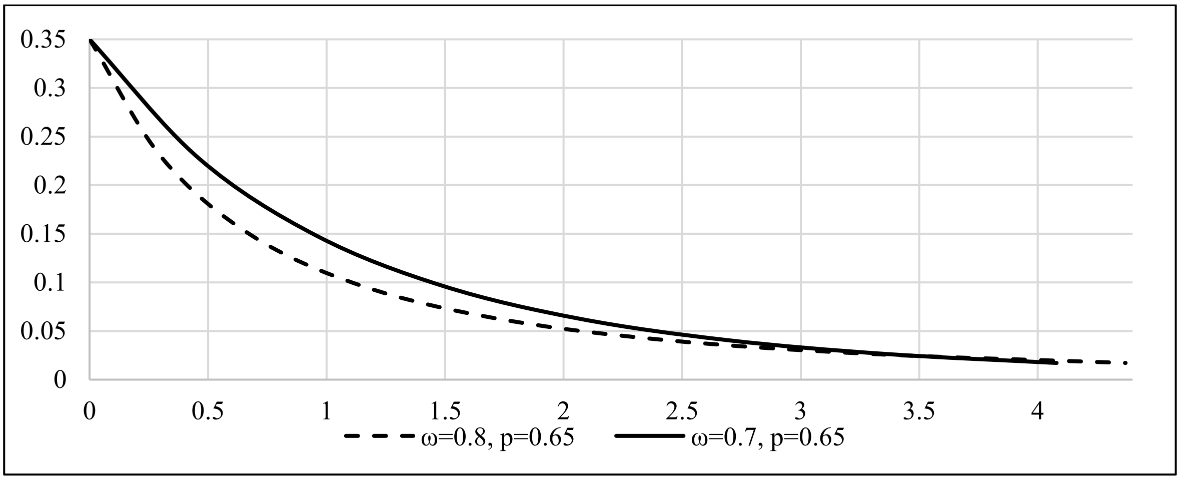
for children of (wl) parents and grandparents, with proba $(1-\mathrm{p}) \cdot \mathrm{p}^2$.

$$\mathrm{z}_{\mathrm{k}+1} = \frac{1-\omega}{\mathrm{p}} + \frac{\omega}{\mathrm{p}} \cdot \mathrm{z}_{\mathrm{k}} \tag{16.4}$$

for children of (wl) ancestors for $k+1$ generation.

The binomial random tastes described by the group of equations (16) can be used to simulate the steady-state distribution of the normalized individual wealth (z). This is presented in the subsequent figure.

Figure 1: Pareto Survival Function



Source:

On the horizontal axis we have the simulated values of (z) and on the vertical the corresponding probabilities (p). Reasonably, the simulations share the same initial point ($z = 0$, $(1 - p) = 0.35$) which corresponds to the initial equation (16.1) which gives the same result, since $p = 0.65$ for both. Furthermore, the shape of the curves indicates that the steady-state wealth distribution is a Pareto probability distribution of type I. The chart represents two Pareto "survival" or "tail" functions which were calculated for $\omega = 0.8$ (dashed curve) and $\omega = 0.7$ (solid curve). The curves estimate the probability that z will take a value greater than a particular z_k. In mathematical notation, this can be written as: $1 - \Phi(z_k) = \text{Proba } (z \geq z_k)$. It can be proved that this functional form holds for any combination of ω, p provided that $\omega < 1$. The latter is the convergence condition of the model as indicated above (see equation (14)). We know that the Pareto "tail function" has the following formula:

$$1 - \Phi(z_k) = \left(\frac{\lambda}{z}\right)^d \tag{17}$$

Furthermore, for $\omega > p$ the following relations hold (Piketty and Zuckman, 2015):

$$\lambda = \frac{1-\omega}{\omega-p} \tag{18.1}$$

and

$$d = \frac{\ln\left(\frac{1}{p}\right)}{\ln\left(\frac{\omega}{p}\right)} \tag{18.2}$$

Equation (17), together with equations (18.1) and (18.2), indicates that as $\omega \to 1$ the parameter (d) will decline. This means that wealth will be concentrated in fewer hands. The simulation chart picks this result, the "survival function" with the higher ω (the dotted curve) has a fatter "tail" than the function with the lower ω (the solid curve). To understand what this means based on the figures of our simulation the average normalized income above say a value of 2 (i.e., $z > 2$) is calculated bellow for $\omega = 0.7$, and $\omega = 0.8$, respectively.

$$a_1 = \frac{\ln\left(\frac{1}{0.65}\right)}{\ln\left(\frac{0.7}{0.65}\right)} = 5.8 \quad \text{and} \quad \overline{z_1} = \frac{5.81}{5.81-1} \cdot 2 = 2.41 \tag{19.1}$$

$$a_2 = \frac{\ln\left(\frac{1}{0.65}\right)}{\ln\left(\frac{0.8}{0.65}\right)} = 2.074 \quad \text{and} \quad \overline{z_2} = \frac{2.074}{2.074-1} \cdot 2 = 3.86 \qquad (19.2)$$

Equations (19.1) and (19.2) tell us that for $\omega = 0.8$ instead of 0.7, the average normalized wealth over 2 increases from 2.41 to 3.86. In other words, when $\omega = 0.8$ people with normalized wealth more than two (2) will hold wealth almost four (3.86) times the average. When $\omega = 0.7$ this figure is less than two and a half (2.41). The latter indicates that small changes in ω bring important changes in the concentration of wealth at the steady-state. This property is attributed to Piketty's "third fundamental equation", where the parameter $\omega = s' \cdot \frac{1+r^e}{1+g}$ is approximated by:

$$\omega = s \cdot e^{(r-g)\cdot H} \qquad (20)$$

Where H represents the time period, i.e., a generation lasting 30 years. If r and g are considered instantaneous values, then equation (20) becomes an equation which shows that the greater the gap r-g, the greater the value of ω and the greater the concentration of wealth in fewer hands as elaborated above, in equations (19.1) and (19.2). From this last point comes the main policy suggestion of Piketty. If we introduce a flat tax on capital income, then r is reduced to its' after-tax value, $\bar{r} = (1-t) \cdot r$. Consequently, the difference between r and g (which is also net of depreciation) is reduced by $t \cdot r$ tempering the unequal distribution of wealth.

The above concludes the core of Piketty's argument on inequality. His reasoning is based on the Solow (1956) model and this has several deficiencies as pointed out already. Some of these issues will be elaborated further in the remainder of the paper. However, I must emphasize at this stage that the analysis includes important contradictions. Although, Piketty considers the initial endowment of wealth as the crucial factor determining income distribution when it comes to dynamics of the differentials of per capita incomes between countries it plays no part provided that the credit market is efficient. This does not make sense, if international credit can bridge the gaps in economic development, it can have a similar effect with income differentials. To put it differently, if inequality can be tempered by efficient credit markets then the analysis so far is not of much relevance.

Moreover, Piketty's definition of capital is highly problematic. It is not only the fact that it is different from the definition of capital in Marx (Marx, 1887, p. 543; Galbraith, 2014; Morgan, 2015) but it is also diverse from the neoclassical definition. In the latter capital is simply the means of production. This heterogeneous set must be consolidated in a homogenous index in order to function as an input in the aggregate production function. Whether this is feasible or not has been a part of a heated debate in the context of the "capital controversy". However, even neoclassical economists would be very reluctant to argue in favor of a "homogeneous capital index" that includes real estate, government buildings, and all sorts of consumer durables like the one introduced and applied by Piketty. For example, when attempting to evaluate the aggregate production function empirically Solow (1957, p. 314) expressly excluded farmland, government, and consumer durables from the measurement of capital. Therefore, Piketty's reliance on neoclassical theory does not suffer only in terms of relevance but has also issues of internal consistency between the analytical categories defined by the theory and those applied by Piketty.

III. Neoclassical Economics and Piketty

The primary concern of neoclassical economists was to disprove Piketty's assertion that inequality is inherent in capitalism. To this end, they forwarded interesting theoretical arguments and empirical measurements as briefly outlined below. However, I must point out from the beginning that their critique is undermined from lack of a theory of their own that can explain rising inequality.

When Solow's paper came out in 1957 it was not greeted with enthusiasm by all mainstream economists. The main reason was that it did not answer whether the steady-state growth path was optimal in the neoclassical sense. An important part of this question had to do with the constant net rate of savings appearing in the model. Mainstream economists were uncomfortable with the notion that the rate of savings was independent of the rate of growth and the marginal product of capital. This is the reason that in the textbook version of the Solow model net savings and growth rates were replaced by the gross measures. Comparing the evolution of the rate of savings and consequently the capital/income ratio ($\beta = s/g$) for gross and net measures is the basis of the critique of Krussel and Smith (2015) on Piketty's "second law of capitalism". They point out that for the textbook model the variable net savings rate falls to zero when growth is zero. However, if we assume like Solow (1957) and Piketty after him that the net savings rate is fixed and the gross savings is endogenous the picture is altered. The following equation determines gross savings (Krussel & Smith, 2015):

$$s(g) = \frac{s \cdot (g + dep)}{g + s \cdot dep} \tag{21}$$

Where: s(g) is the gross savings rate and *dep* the rate of depreciation. From equation (21) it is obvious that with a fixed net rate of savings when growth falls to zero then the gross rate of savings will become equal to 1. For the neoclassical story, where investment, savings, and the accumulation of capital stock are identical, this is not reasonable. Moreover, empirical evaluations included in Krussel and Smith (2015) show that the fixed net savings model performs poorly in explaining the savings patterns. Consequently, they argue that Piketty's explanation of a rising β and more importantly his predictions for further increases in this ratio, due to constant net savings and declining growth rates, do not stand.

However, and irrespective of the above both the capital income ratio and the profit share have increased in the neoliberal era and this requires an explanation. Krussel and Smith (2015) make a clever point by suggesting that this was due to capital gains but say nothing more. They conclude that inequality is an important aspect of contemporary capitalism which requires explanation and hope that testing the explanatory power of Piketty's "third fundamental law of capitalism" (i.e., $r - g > 0$) will give better results.

The last statement is kind of short-sighted. One does not need to be an expert on neoclassical growth theory to understand that with a variable net savings rate Piketty's analytical reasoning and policy suggestions do not stand. The latter is the key argument of the critique of Gregory Mankiw (2015). His main question is: What is the optimal tax rate on capital income and wealth that minimizes inequality? Using a simple model where the rate of growth depends on the net rate of return on capital and standard optimization techniques he arrives at the usual neoclassical conclusion (Chamley, 1986; Judd, 1985, Atkeson, Chari & Kehoe, 1999). If capital is taxed accumulation will weaken (net savings will drop) and wages which are the difference between output and return on capital (profits) will fall. In the end, inequality increases although workers will consume the proceeds from capital taxation. So, Mankiw concludes, that the best for workers' incomes is no taxes on capital. As Piketty (2015) admits this is the case for a neoclassical model with infinite elasticity of accumulation (net savings) on the net rate of return.

But like with Krussel and Smith (2015), the problem remains, inequality is rising in contemporary capitalism. In this regard, Mankiw (2015) is not much help. He concludes his paper by suggesting apologetically that it is better to be in the lower-income bracket in a rich and growing economy with high inequality than a middle income in a lower inequality roughly stagnant economy. Of course, this is not an explanation of the dynamics of inequality.

In general, the proponents of the "conditional convergence" Solow model (Mankiw et. al., 1992) focus on the implications of constant net savings in order to criticize Piketty's analytical reasoning and policy suggestions. The advocates of endogenous technology growth models (Lucas, 1988), on the other hand, criticize the exogenous technology and the absence of institutional and political factors from Piketty's reasoning (Acemoglu & Robinson, 2015). However, Acemoglu and Robinson (2015) who are the most prominent critics of Piketty coming from this line of thought, do not attribute these shortcomings to the Solow model, but some connection of Piketty to Ricardo and Marx. The latter is not accurate, as I will show in the next section with emphasis on Marx. For now, I will outline their critique on Piketty.

They begin by checking the correlation of growth and the bond rate minus the growth rate with inequality. Although, the association of the rate of profit with the long-term bond rate is good only for the neoclassical theory their calculation is consistent with Piketty's data[7]. Therefore, the disappointing results they find are rightfully considered as denying the association of inequality with the "third fundamental law of capitalism" ($r - g > 0$). Having weakened Piketty's argument Acemoglu and Robinson then move to establish their own argument for explaining inequality. On the empirical level, their idea is that the dynamics of the income share of the top 1% can be misleading in understanding the overall income distribution. They try to make their case not by looking into the income distribution in each country but by comparing the income distribution in two extreme cases. Namely, the social-democratic Sweden and South Africa which suffered for many years under the racist apartheid regime.

They argue that the top 1% of the South African population cannot be representative of the income distribution of the whole population and its' evolution in the pre- and post-apartheid

[7] It is worthy to keep in mind that Piketty due his definition of capital underestimates the profit rate and finds a value around 5% on average which is close to the average long-term bond rate.

era. Obviously, this is not a valid argument not only because the country of reference is an extreme case but more importantly because recent econometric work explains the connection of the distributions of the bottom 97-99% with the top 1-3% of incomes. Specifically, it has been proved that the bottom 97% of incomes, mainly labor incomes, follow the exponential distribution while the top 1-3% consisting of capital incomes follows the Pareto distribution (Dragulescu & Yakovenko, 2002; Silva & Yakovenko, 2004; Yakovenko, 2007). Moreover, the two distributions can be combined to derive a Gini coefficient for the overall income distribution (Dragulescu & Yakovenko, 2001; Silva & Yakovenko, 2004). The overall coefficient depends on the linear relation of the Gini measurement for the exponential distribution (bottom 97%) and the top incomes share. This way the impact of wage differentials and the top 1% capital incomes (on inequality) are identified separately. The analysis can be refined further to identify the sources of the increase in top capital incomes, for example, the lower wage share, capital gains, etc. (Shaikh, 2016). In other words, we can identify the sources of inequality as I will discuss further in the next section.

However, Acemoglu and Robinson (2015) do not provide any such empirical evidence. They simply consider that they can explain inequality by conceptualizing the primary impact of political institutions that influence economic institutions, which in turn influence technology, skills and prices, and finally economic performance and inequality (Acemoglu, Johnson & Robinson, 2005; Acemoglu and Robinson 2015). Then they turn back to the South Africa-Sweden example to support their causality chain with historical facts. The result is disappointing, they admit that their analytical reasoning can explain the distribution of labor incomes (the bottom 97%) but not the evolution of the top 1%-3%. This is no surprise since top incomes follow a common pattern in almost all countries. What is kind of surprising is that Acemoglu and Robinson treat the two distributions as independent from each other. In other words, they analyze income differentials as if the top income bracket does not affect the overall inequality. On these grounds, they conclude that there is no theory of inequality since labor income differentials have a different history in each country depending on local institutions.

Acemoglu and Robinson attempt to limit the discussion on inequality to meritocracy in order to exclude top incomes from the analysis. The idea is that if Warren Buffet, for example, becomes richer this will be at the expense of other rich people and does not affect meritocracy in the American society (Acemoglu & Robinson, 2015). What the society should focus on is the effect of political institutions on economic institutions supporting vested interests that result in low and middle incomes differentials. The latter can be removed with the appropriate institutional reform. Neoclassical economists systematically forget that the share of the top 1% on aggregate income and wealth has increased substantially in the neoliberal era. This means that it is more likely that rich people become richer at the expense of the whole society and not just other rich people.

Nevertheless, institutions play an important part in income distribution. In the classical political economy, they are the result of socio-economic conditions, as we will discuss next.

IV. Marx and Piketty

So far, we have considered growth and distribution literature extending for almost 70 years. In this long reference, we did not come across a single paper which associates wages and the wage share with unemployment. The reason is that all the literature considered so far relies on the neoclassical theory where the wage is a market-clearing price. Therefore, as indicated also by the production functions discussed above, the economy is assumed to operate at or around full employment. In classical political economy and especially Marx this is not the case. Structural unemployment and class struggle play an important part in income distribution and inequality.

To understand how this happens we need to outline the core argument. In Marx, capitalist production relations push towards the constant mechanization of production because it brings greater control on labor for the capitalist. Mechanization facilitates the intensification of the production process, increases productivity, and raises the rate of (surplus value) exploitation of labor. Therefore, contrary to the assertions of Piketty (2014, p. 10) Marx expected "durable technical progress". Moreover, technology is endogenous in Marx. The contention of certain neoclassical economists that Marx "ignored the endogenous evolution of technology" (Acemoglu & Robinson, 2015) is simply wrong. The ratio of "constant to variable capital" which is the equivalent of the capital/labor[8] ratio in Marx is neither fixed nor moves towards a steady-state amount imposed by the neoclassical production function. It is expected to be constantly increasing. In other words, contrary to Piketty (2014, p. 228) and Solow (1957) before him, in Marx technical change is not neutral.

[8] I use the word equivalent because in Marx the ratio of dead to living labour referred to as the "value composition of capital" and denoted by (c/v) is calculated in (labour) value and not physical units like the capital labour ratio (K/L) in the neoclassical production function.

However, the process is not harmonious but contradictory. Although the more mechanized techniques have lower production costs, they involve higher investment costs, the productive "powers of labor must be paid for" as Marx (1973, p. 776) states. In other words, these techniques have a higher ratio of (fixed) capital per unit of output or a higher β (in a closed economy) in Piketty's terms[9]. Therefore, if we follow Marx, we anticipate an ever-increasing β and this has nothing to do with the rate of savings relative to the rate of growth.

Ruthlessly summarizing, the higher investment costs ensure that the rate of profit will eventually fall as indicated by the following equation.

$$r = \frac{P}{K} = \frac{Y}{K} \cdot \frac{P}{Y} = \frac{\alpha}{\beta} \tag{22}$$

Where P is total profits and total income Y is assumed to equal output. Equation (22) looks like a rearrangement of Piketty's "first fundamental law of capitalism" (see equation (4)). However, both the analytical categories (capital, profit, and income/output) and the economic interpretation are completely different. Equation (22) says that because the profit share α, determined by unemployment and class struggle, has an upper limit equal to 1[10] whereas the capital income ratio β is expected to endlessly increase, due to persistent mechanization, the rate of profit will tend to fall (Rosdolsky, 1977, pp. 398-413). This is the fundamental law of capitalism in Marx. As Marx himself admits it is "one of the most striking phenomena of modern production" since it suggests that the strive of individual capitalists for greater profits ends, against their will, to a lower rate of profit. However, the theory explains how this happens. It is realized through competition, which in classical political economy and Marx has nothing to do with perfect competition. Capitalist firms irrespective of number and size are active price cutters and not passive price takers like the neoclassical firm. This means that capitalists will undertake lower profit rate techniques if they enable them to penetrate the market share of their competitors. It can be shown that the latter is a sufficient condition for the capital/output ratio to increase and the profit rate to fall. Therefore, the assertion of Acemoglu and Robinson (2015), suggesting that Marx believed that competition would decline in the process of capitalist accumulation due to capital concentration, is also false. They imply that Marx follows

[9] It must be mentioned that the definition of capital is much different between the two approaches.
[10] When $\alpha = 1$ we have the unrealistic case where the whole income is profit.

the theory of perfect competition and its' dark side neoclassical monopoly and this is not correct.

The rate of profit and its dynamics is the key variable of capitalist economies. It determines growth, employment, and influences income distribution. Numerous empirical works in many countries confirm that, with a proper definition of the Marxian categories of profit and capital, the rate of profit has a downward tendency. The declining rate of profit is the cause of long waves in production that appear almost periodically every 40 years throughout the history of capitalism (Kontratieff, 1984; Shaikh, 2016, p. 766). However, contrary to the assertions of Piketty (2014, p. 9) there is not a single quote from Marx which suggests that the law of the falling rate of profit will bring the end of capitalism. From his early works (Marx & Engels, 1848), Marx considers the overthrow of capitalism as a political act and not an economic law.

Having clarified the above we can now move to the operation of the law of the falling rate of profit on employment and income distribution. The argument is that due to the mechanization of production constant capital grows faster than variable capital. On the one hand, this increases labor productivity and on the other creates a surplus population at normal capacity utilization (Marx, 1887, The Capital Vol. I, Ch.25). The surplus population expands and contracts according to all sorts of economic fluctuations, i.e., both industrial cycles and long waves (Marx, 1887, Capital Vol. I, Ch.25; 1894, Capital Vol. III, Ch.14). These fluctuations in the "reserve army" regulate the dynamics of wages at the prevailing institutional context which reflects the level of the class struggle.

One of the first and most celebrated formulations of a good part of these ideas in a mathematical model is the Goodwin (1967) model. Due to its' path braking prey-predator formulation, it results in a recurrent growth cycle which depends on the fluctuations of the wage share. The model is based on a real wage share "Philips curve" with an equilibrium unemployment rate. The idea is that as accumulation accelerates unemployment drops, the wage share increases, the profit rate, which is also the rate of accumulation, falls, and as a result growth fades. Consequently, unemployment increases, the wage share drops, profitability is restored, and the process repeats itself.

Although the model confirms the existence of a persistent "reserve army of labor" like in Marx it has various deficiencies. The most important is that because the "natural rate of growth" is exogenous and constant[11], class struggle is ineffective, as in Piketty. The only difference is that in Goodwin it leads to a higher equilibrium unemployment rate without altering the equilibrium wage share.

This is not the case in Marx where, contrary to neoclassical economics, the wage has a social and historical component (Dobb, 1973, pp. 91-92, pp. 152-153). However, the historical component represents a minimum wage desired by capital, whereas labor pushes wages away from the minimum towards an upper limit. The latter depends on the impact of wages on profits and the viability of the firm (Botwinick, 1993). This means also, that, contrary to Ricardo, Marx did not consider wages as a social constant that can become an obstacle to capital accumulation (Marx, 1875). The idea is that higher wages will attract additional labor, through immigration, or labor will be released through endogenous productivity growth limiting the effectiveness of labor unions' bargaining power.

The above explains why Acemoglu and Robinson (2015) are again wrong to believe that the mature Marx considered some "iron law of wages" or a permanent downward trend in the wage share as a "law" of capitalism. The brochure *The Critique of the Gotha Programme* (Marx, 1875), referred also above, is devoted to attacking these ideas. Marx dedicated a good part of his political activity in fighting for the eight-hour working day to reduce labor exploitation[12]. The latter proves that he considered that institutions play an important part in the intensity of inequality.

These ideas can be formulated in terms of contemporary economic analysis (Shaikh, 2016, pp. 685-702) and are briefly outlined here below. We can define productivity as the sum of the real

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[11] This means that productivity (d) and population growth (n) are model parameters. The Goodwin (1967) model solution arrives to the following system of non-linear differential equations:

$$\frac{\dot{sw}}{sw} = h \cdot (v_l - v_l^*) - d \quad \text{and} \quad \frac{\dot{v_l}}{v_l} = (1 - sw) \cdot \frac{1}{\beta} - (n + d) = \frac{\alpha}{\beta} - (n + d)$$

The first equation is the Goodwin version of equation (25) with constant productivity growth (*d*) and a linear wage growth employment function $\frac{\dot{wr}}{wr} = h \cdot (v_l - v_l^*)$. The parameter *h* is the reaction parameter of wages to an increase in employment and the employment rate v_l^* is the rate of employment above which wages and the wage share begins to rise. A high value of *h* and a low value v_l^* are indications of labour class strength. The equilibrium value of employment is given from the first equation and is: $v_l^e = v_l^* + \frac{d}{h}$. This means that high labour strength means lower employment. At the same time, the equilibrium wage share given from the second equation is: $sw^e = 1 - (n + d) \cdot \beta$ (β is assumed constant) which is not affected by labour strength.

[12] The eight-hour working day was the central political objective of the 1st International.

wage (*wr*) plus the real profit per worker. In turn, productivity depends on technology and the length of the working day. On these grounds, the real wage can be written as the product of the current "social climate" variable *m* and productivity *y*. This can be formalised as:

$$\mathrm{wr_t} = \mathrm{m_t} \cdot \mathrm{y_t} \tag{23}$$

And in continuous time:

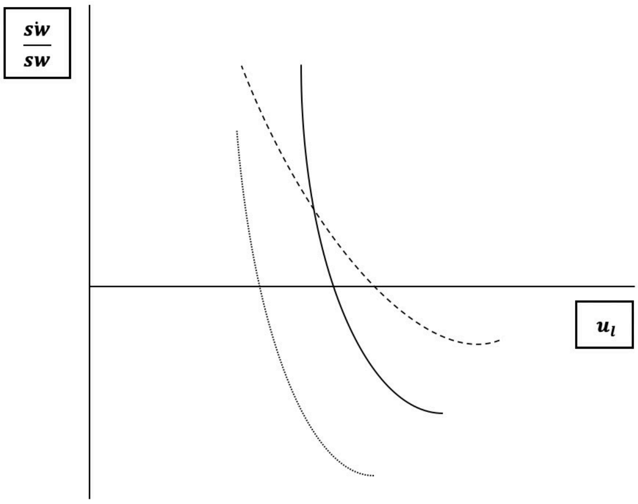
$$\frac{\dot{\mathrm{wr}}}{\mathrm{wr}} = \frac{\dot{\mathrm{m}}}{\mathrm{m}} + \frac{\dot{\mathrm{y}}}{\mathrm{y}} \tag{24}$$

This analysis of wages can be extended to inequality. Abstracting from value transfers we can write the wage share (*sw*) as the ratio of the real wage (*wr*) to the productivity (*y*). From equations (23) and (24) it is not difficult to figure out that the social climate variable (*m*) represents the wage share. Given the previous discussion, we can consider the rate of growth of the wage share a function of the rate of unemployment.

$$\frac{\dot{\mathrm{sw}}}{\mathrm{sw}} = \frac{\dot{\mathrm{m}}}{\mathrm{m}} = \frac{\dot{\mathrm{wr}}}{\mathrm{wr}} - \frac{\dot{\mathrm{y}}}{\mathrm{y}} = \mathrm{f}(\mathrm{u_l} - \mathrm{u_l^*}), \text{ where } \mathrm{sw} = \frac{\mathrm{wr}}{\mathrm{y}}, \text{ and } f' < 0 \tag{25}$$

Equation (25) points to a real wage share Philips curve. Some possible examples are presented in the figure that follows.

Figure 2:



Source:

Any change in the reactive strength of labor will result in a parallel shift of the curve (compare the solid and dotted lines). This means that wages change at a different rate at any level of unemployment. A counter - clockwise shift on the other hand indicates an increase in the relative strength of labor (compare the solid and dashed lines).

Booms and busts move the unemployment rate up and respectively down the same curve. Abolishing labor supporting institutions shifts the curve downwards while reducing the equilibrium wage share and the normal rate of unemployment. So, if we look for inequality in the neoliberal era under this theoretical perspective, we should look for the impact of a downward shift in the real wage share Philips curve.

But this is not the whole story. In Marx besides the profit from production, analyzed so far with emphasis on distribution, there is also a second source of profit. It is what he calls "profit upon alienation" and refers to transfers of value between the circuit of revenue and the circuit of capital or between current and capital cost accounts. Examples are the sale of an asset (say a building, or corporate stock purchased in the past) and the interest paid on household debt. This type of income/profit increased greatly during the neoliberal era due to the deregulation of the financial markets. In the accounts of the Internal Revenue Service (IRS) it is recorded as interest payments, capital gains, and in certain cases asset appreciation. It reflects significant flows of interest from labor incomes, due to the increased borrowing of households, as well as "capital gains" for the top 1%.

Shaikh (2016) evaluated empirically the reduction of the wage share and the rise of actual and fictitious financial profit as the sources of the rising inequality in the United States[13]. Using the techniques of Yakovenko et. al. (2002; 2004; 2007), briefly outlined in the previous section, he found that the rise in the incomes of the top 1% is responsible for the almost 30% increase of the Gini coefficient in the neoliberal era. Moreover, the two factors (financial incomes and the profit share) almost doubled the ratio of property income to personal income. Regressing

[13] Although additional empirical investigation is required, it is almost certain that the main source of rising inequality in the neoliberal era is "within-country" inequality. It is for this reason that I confined the Marxist analysis to the "wage share" and the "profit upon alienation". Of course in the Marxist tradition there is extensive literature explaining inequality through transfers of value between the developed and underdeveloped world. Recent empirical studies are supportive of this literature as well as the approach taken herein. They show high, persistent but constant cross-country inequality throughout the greatest part of the neoliberal era (Lakner & Milanovic, 2015).

the ratio of property/to-personal-income ratio on the Gini coefficient explains 82% of its' variation ($R^2 = 0.8171$).

V. Conclusion

Piketty's reliance on the early neoclassical growth theory undermines his analytical reasoning on inequality. This explains why he cannot explicate its' persistent increase over the last thirty years. As we saw in the second section, his models describe at best a steady-state equilibrium with a high profit share and concentration of wealth. Moreover, his analysis of income inequality contradicts his discussion on global inequality. The former depends on the difference $r - g > 0$ and the consequent concentration of wealth while the latter on the inefficiency of the credit market. It is obvious that if efficient credit markets can lead to global convergence, they can reduce also domestic inequality. In this case, inequality is not due to a "law" of capitalism but to market imperfections. Moreover, Piketty's definition of capital contradicts the traditional measures applied for the calculation of the factor "capital" by mainstream economists. In other words, the analytical categories applied by Piketty are not consistent with the Solow growth model he applies in order to explore income distribution (Solow, 1957; Galbraith, 2014).

Neoclassical economists (Acemoglou & Robinson, 2015) have tried to attribute these deficiencies to some influence of classical political economy and especially Marx on Piketty. They suggest that both Marx and Piketty rely on exogenous technical change and place no weight on the impact of institutions. This may be true for Piketty and his reliance on the Solow (1956) model. But, as discussed in fourth section, in Marx, technology is endogenous stemming from capitalist production relations and resulting in the law of the tendency of the rate of profit to fall. The latter is the outcome of the constant mechanization of production which brings about a persistent pool of unemployed, a "reserve army of labor" in Marx's words. The fluctuation in this "army" regulates the wage share in the prevailing institutional environment and the phase of the business cycle. The latter implies that class struggle plays an important part in determining the wage share and the normal rate of unemployment (Botwinick, 1993). These theoretical elaborations can explain a good piece of the rise of inequality during the neoliberal era, through the deregulation of the labor market and the abolition of the welfare state. It has been shown empirically (Shaikh, 2016) that the drop in the real wage share (rise in the profit share) is one of two main reasons for the increase in inequality following 1980. The other is the capital gains and interest income realized by the top 1% during the same period, resulting from the deregulation of the financial markets. The above indicate that contrary to Piketty under the classical/Marxian analysis labor can regain what it has lost after 1980 through class struggle.

The above does not imply that inequality can be eliminated in capitalism for the Marxist school. Besides the permanent impact of the "reserve army of labor" outlined above the "initial endowment" also prevents this from happening. The conditions of the "primitive accumulation of capital" influence the development of countries, in the long run, preventing convergence. In the same fashion inheritance influences "within-country" social mobility. Therefore, the class struggle in capitalism can temper but cannot remove inequality.

This whole debate raises a broader issue. The tactic of certain neoclassical economists in attacking Piketty in order to get rid of the whole discussion on inequality (see third section) cannot go on. Even mainstream economists (Naidu, Rodrik & Zucman, 2020) acknowledge the need for a shift in the profession towards the great problems of our time where inequality has a prominent place. This is hopeful and challenging for economics in the new century.

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