Continuous Phase Modulation

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A class of jointly power and bandwidth efficient digital modulation schemes with constant amplitude

Minimum Shift Keying (MSK) [1] or Fast Frequency Shift Keying (FFSK) is a digital modulation technique with constant amplitude which has been studied extensively during recent years. The properties of MSK are now understood [2-5]. In this article, we report on methods to improve on MSK while maintaining a constant amplitude. By improvement we mean a narrower power spectrum, lower spectral sidelobes, better error probability, or all of the above. The cost of signal processing and the speed and complexity with which it can be performed has steadily been improved over recent years. Further improvements are to be expected. Therefore, it is reasonable to study what can be accomplished with methods that might be too complex for cost effective realization today, but perhaps may be easily achievable tomorrow.

In this paper, we consider a number of methods for constructing constant amplitude signals which significantly outperform MSK [5-103]. We also discuss at what level of complexity these improvements are obtained and realized.

A Class of Constant Amplitude Signals

A large class of constant amplitude modulation schemes is defined by:

$$s(t) = \sqrt{\frac{2E}{T}}\, cos(2\pi f_0 t + \phi(t,\underline{\alpha})) \qquad (1)$$

where the transmitted information is contained in the phase:

$$\phi(t,\underline{\alpha}) = 2\pi h \sum_{i=-\infty}^{\infty} \alpha_i q(t-iT) \qquad (2)$$

with $q(t) = \int_{-\infty}^{t} g(\tau)d\tau$. Normally the function $g(t)$ is a smooth pulse shape over a finite time interval $0 \leq t \leq LT$ and zero outside. Thus L is the length of the pulse (per unit T) and T is the symbol time. E is the energy per symbol, f_0 is the carrier frequency and h is the modulation index. The M-ary data symbols α_i take values $\pm 1, \pm 3 \cdots \pm (M-1)$. M is normally a power of 2. Below we will mainly consider binary (M=2), quaternary (M=4) and octal (M=8) systems. From the definition of the above class of constant amplitude modulation schemes we observe that the pulse $g(t)$ is defined in instantaneous frequency and its integral $q(t)$ is the phase response. The shape of $g(t)$ determines the smoothness of the transmitted information carrying phase. The rate of change of the phase (or instantaneous frequency) is proportional to the parameter h, which is normally referred to as the modulation index. Above we have normalized the pulse shape $g(t)$ in such a way that $\int_{-\infty}^{\infty} g(t)dt$ is 1/2. This means that for schemes with positive pulses of finite length, the maximum phase change over any symbol interval is $(M-1)h\pi$.

Thus, by choosing different pulses $g(t)$ and varying the parameters h and M, a great variety of CPM schemes can

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be obtained. Some of the more popular pulse shapes during recent years are listed in Table I, such as Continuous Phase Frequency Shift Keying (CPFSK) [6-8] Tamed Frequency Modulation (TFM) [17], Generalized TFM (GTFM) [23], Gaussian MSK (GMSK) [19], Duobinary FSK (2REC) [21], Raised Cosine (LRC) [13,14] and Spectrally Raised Cosine (LSRC) [13,14]. In Table I we use the notation LRC for a raised cosine pulse of length L symbol intervals. Thus, 3RC is a raised cosine pulse of length $3T$. For spectral raised cosine, LSRC, the main time lobe has width LT, see the definition in Table I. The rectangular pulse of length L is denoted LREC. The pulse of length $1T$, that is 1REC, is referred to as CPFSK in the literature. The 2REC pulse with length $2T$ is also called duobinary.

It is convenient to view CPM as a generalization of Minimum Shift Keying (MSK) which is obtained as a special case of the signals defined in (1) by selecting the pulse 1REC (L=1) from Table I and using binary (M=2) data with $h = 1/2$. A detailed description of MSK is given in [2]. The CPM signal can be viewed as both phase modulation and frequency modulation. For understanding the optimum coherent receiver, it is advantageous to view the signal as phase modulation.

TABLE I
Definition of the frequency pulse functions $g(t)$ used in this paper. By varying a, b and $g_0(t)$, the class of GTFM pulses is obtained.

LRC	$g(t) = \begin{cases} \frac{1}{2LT}[1 - \cos\frac{2\pi t}{LT})] & ; 0 \leq t \leq LT \\ 0 & ; otherwise \end{cases}$ L is the pulse length, e.g., 3RC has L=3.
TFM	$g(t) = \frac{1}{8}\left[ag_0(t-T) + bg_0(t) + ag_0(t+T)\right]$ $; a = 1; b = 2$ $g_0(t) \approx \sin\left(\frac{\pi t}{T}\right)\left[\frac{1}{\pi t} - \frac{2 - \frac{2\pi t}{T}\cot\left(\frac{\pi t}{T}\right) - \frac{\pi^2 t^2}{T^2}}{\frac{24\,\pi t^3}{T^2}}\right]$
LSRC	$g(t) = \frac{1}{LT}\cdot\frac{\sin\left(\frac{2\pi t}{LT}\right)}{\frac{2\pi t}{LT}}\cdot\frac{\cos\left(\beta\cdot\frac{2\pi t}{LT}\right)}{1-\left(\frac{4\beta}{LT}\cdot t\right)^2} \; ; 0 \leq \beta \leq 1$
GMSK	$g(t) = \frac{1}{2T}\left[Q\left(2\pi B_b\frac{t-\frac{T}{2}}{\sqrt{\ell n 2}}\right) - Q\left(2\pi B_b\frac{t+\frac{T}{2}}{\sqrt{\ell n 2}}\right)\right]$ $; 0 \leq B_bT < \infty$ $Q(t) = \int_t^\infty \frac{1}{\sqrt{2\pi}} e^{-\tau^2/2}\, d\tau$
LREC	$g(t) = \begin{cases} \frac{1}{2LT} & ; 0 \leq t \leq LT \\ 0 & ; otherwise \end{cases}$ L=1 yields 1REC which is most often referred to as CPFSK.

Memory is introduced into the CPM signal by means of the continuous phase. Each information carrying phase function $\phi(t,\underline{\alpha})$ is continuous at all times for all combinations of data symbols. Even when the data symbols are uncorrelated, further memory can be built into the CPM signal by choosing a $g(t)$ pulse with $L > 1$. These schemes have overlapping pulse shaping and are sometimes called partial response techniques. CPM signals with $L \leq 1$ are referred to as full response schemes. In this case all the memory is in the continuous phase.

Although the CPM signals in (1) are in principle conceivable for any value of the modulation index h, a key to the development of practical maximum likelihood detectors is to consider CPM schemes with rational values of h. For $h = 2k/p$ where k and p have no common factors, the phase $\phi(t,\underline{\alpha})$ during interval $nT \leq t \leq (n+1)T$ can be written:

$$\phi(t,\underline{\alpha}) = 2\pi h \sum_{i=n-L+1}^{n} \alpha_i q(t-i\tau) + \theta_n = \theta(t,\underline{\alpha}) + \theta_n \quad (3)$$

where $\theta_n = \left[h\pi \sum_{i=-\infty}^{n-L} \alpha_i\right]$ modulo 2π has only p different values. Thus the total number of states that (at most) is needed to describe the signal in (1) is $S = pM^{(L-1)}$ where a state is defined as the vector $(\theta_n, \alpha_{n-1}, \alpha_{n-2}, \cdots, \alpha_{n-L+1})$. The state vector consists of the phase state θ_n and $M^{(L-1)}$ correlative states for partial response systems. For a full response system the number of states is $S = p$. The significance of the finite state description for certain CPM signals becomes evident in the following sections.

As an example of a CPM scheme we have chosen binary 3RC; the pulse g(t) is a raised cosine pulse of length 3 symbol intervals. Figure 1a shows the pulse g(t) and phase response q(t) for 3RC. For comparison, the corresponding functions are also shown for 1REC (CPFSK). The information carrying phase function $\phi(t,\underline{\alpha})$ from (2) is illustrated both for 1REC and 3RC in Fig. 1b for a particular data sequence. Note that all changes in the 3RC phase takes longer time than for the CPFSK scheme. Figure 1c shows all phase functions starting at the arbitrary phase 0° and time 0 with the two previous data symbols being +1, +1. It is obvious from Fig. 1c that all phase changes are very smooth. The corresponding phase tree for MSK has linear phase changes with sharp corners when data changes. The phase tree in Fig. 1c also displays the state vector at each node.

The finite state description of the CPM signal implies a trellis description and this property becomes evident by using the modulo 2π property of the phase. By plotting $I = \cos[\phi(t,\underline{\alpha})]$ and $Q = \sin[\phi(t,\underline{\alpha})]$ versus time in a three-dimensional plot, all signals appear on the surface of a cylinder. Such a phase cylinder is shown in Fig. 2a

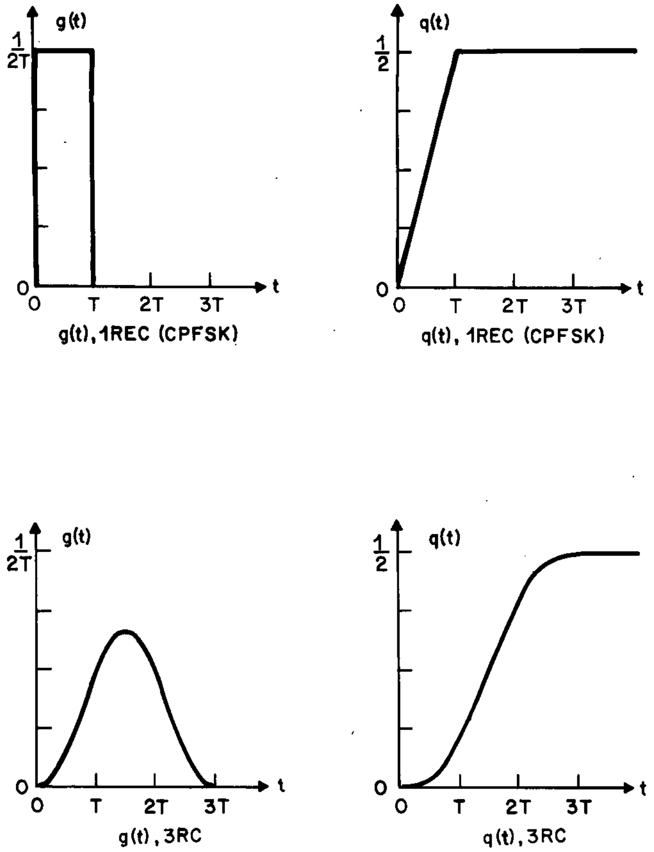

Fig. 1a. Pulse shapes g(t) *(instantaneous frequency) and phase responses* q(t) *for the full response 1REC (CPFSK) and partial response 3RC CPM schemes.*

for the parameters used in Fig. 1c. This scheme has 12 states. Contrary to Fig. 1c where restrictions were imposed for $t < 0$, the phase cylinder shows all signals over three symbol intervals. The phase nodes and some of the state vectors are also shown. To clarify the connection between the tree in Fig. 1c and the trellis in Fig. 2a, we have marked three identical transitions with arrows in both figures.

While the appearance of the phase tree does not change with h, the number of phase states and the phase cylinder do. Figure 2b shows how simple the phase cylinder is for $h = 1/2$ and Fig. 2c shows how the complexity has grown for $h = 4/5$. It is seen in Fig. 2b that with a proper phase offset, I and Q have quite open amplitude eye diagrams. We will see that this leads to simple linear receivers and this is the reason for the popularity of the different binary $h = 1/2$ schemes.

An interesting generalization of the CPM class is to let the modulation index vary cyclically with time. This gives the so called multi-h schemes [24-26]. These systems have better performance than the fixed h schemes. Typically most of the available improvement is obtained with two or three different h-values.

Power Spectra and Bandwidth

There are a large number of methods available in the literature for calculating the power spectrum of CPM [27-38]. Computer simulations can of course also be employed to estimate the power spectra. Bandwidth does not have a unique definition. To illustrate the spectral efficiency of the signal for example, we can define bandwidth as the frequency band around the carrier frequency containing 99 percent of the signal power.

Figure 3 shows the power spectral density of some binary CPM schemes. GMSK4 means that the GMSK pulse in Table I is truncated symmetrically to four symbol intervals. The schemes 3RC, GMSK4 with $B_bT = 0.25$ and 3SRC6 have comparable power spectra. The corresponding pulses $g(t)$ are also quite similar and so are their detection properties. Figure 3 also shows the MSK and the 2REC (duobinary) schemes. Notice the lower spectral sidelobes for the smooth partial response schemes.

For comparison, Fig. 4 shows power spectra for some 4-level schemes ($M=4$). The rate of reduction of the spectral sidelobes is determined by the smoothness of the pulse $g(t)$. For most practical applications, the raised cosine pulses probably have sufficient smoothness.

Figures 3 and 4 also illustrate that a longer pulse $g(t)$ yields narrower power spectra for fixed h and M. TFM

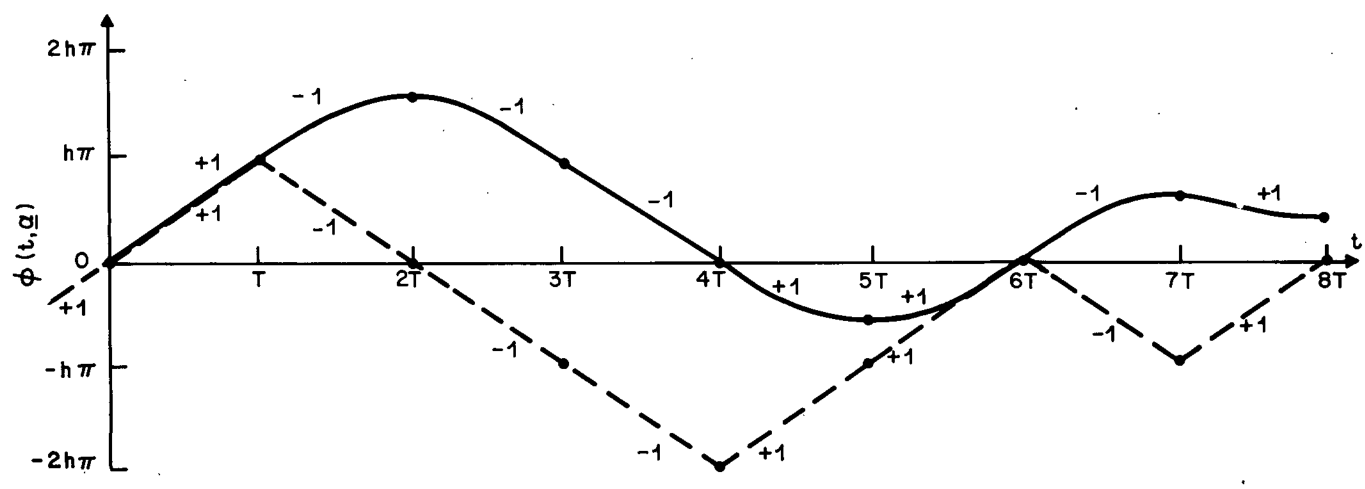

Fig. 1b. Examples of the information carrying phase function $\phi(t,\underline{\alpha})$ *for 1REC (dashed) and 3RC (solid) for the data sequence +1, −1, −1, −1, +1, +1, −1, +1.*

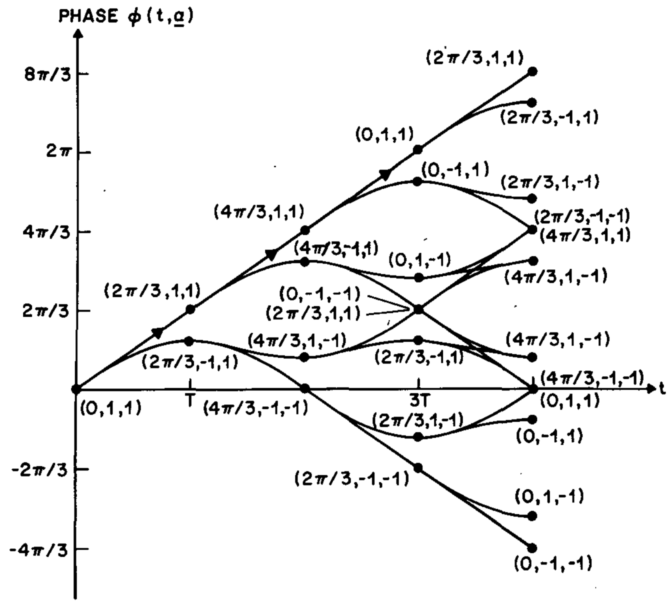

Fig. 1c. Phase tree for binary 3RC with h = 2/3. *The state description of the signal is also given. Notice the transitions with arrows. These are also shown in Figure 2a.*

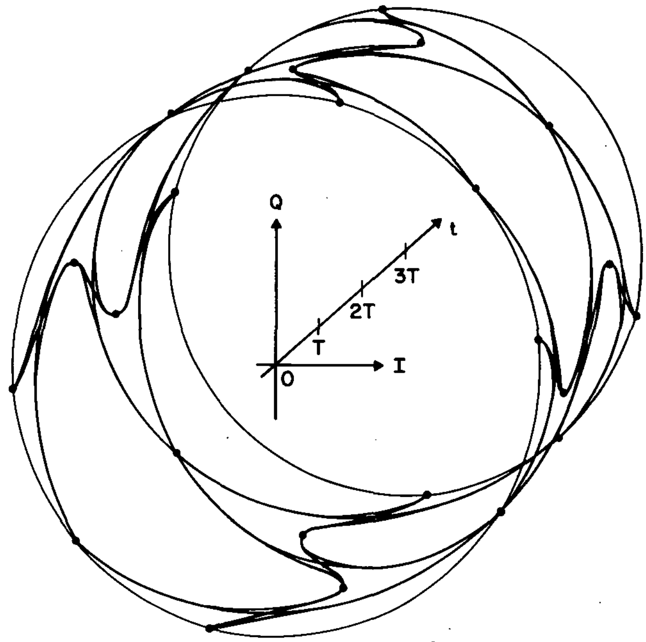

Fig. 2b. Phase cylinder for M = 2, *3RC,* h = *1/2.*

has a power spectra similar to binary 3.7 RC or 3.7 SRC. GMSK with $B_bT = 0.18$ corresponds approximately to 4RC and GMSK with $B_bT = 0.2$ to TFM.

The width of the main spectral lobe decreases with increasing L but increases with increasing h and M. A few results about optimum pulse shapes are given in the literature [22]. The relationship between bandwidth or fractional out of band power and pulse shape $g(t)$ is rather complicated.

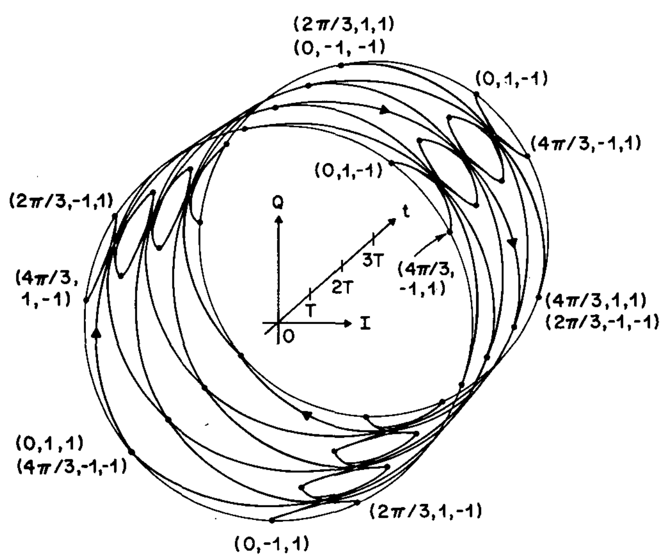

Fig. 2a. Phase-cylinder for M = 2, *3RC with* h = *2/3. Compare the phase tree in Figure 1c. Note the arbitrary phase-offset between the phase in the tree in Figure 1c and the cylinder in Figure 2a. Also note that the tree is plotted over 4T and the cylinder over 3T. Notice the transitions with arrows. These are also shown in the tree in Figure 1c.*

Detection Efficiency and Error Probability

We will now consider the joint power and spectral efficiency of the CPM signals. It can be shown that coherent maximum likelihood sequence detection can be performed for all CPM schemes that can be described by the finite state and trellis description given above. Although the structure of the optimum ideal coherent receiver for CPM is known [5], it is difficult to evaluate its bit error probability performance. Simulations are required for low channel signal-to-noise ratios. The most convenient and useful parameter for describing the error probability of CPM schemes with maximum like-

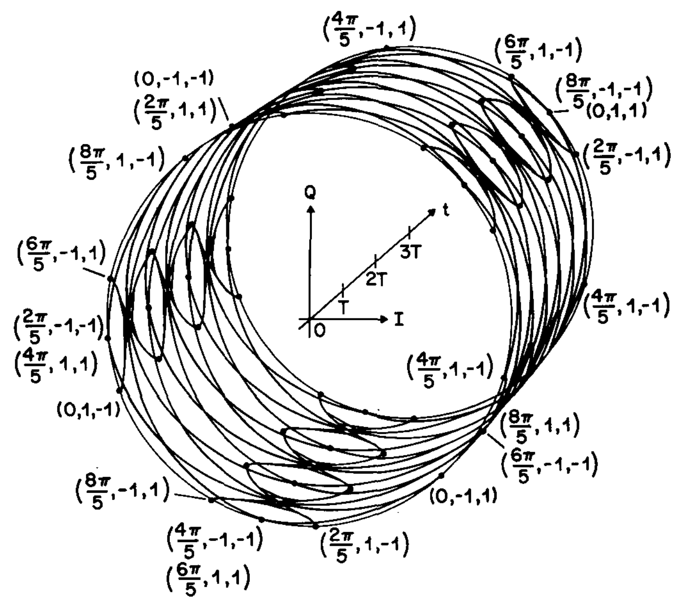

Fig. 2c. Phase cylinder for M = 2, *3RC with* h = *4/5.*

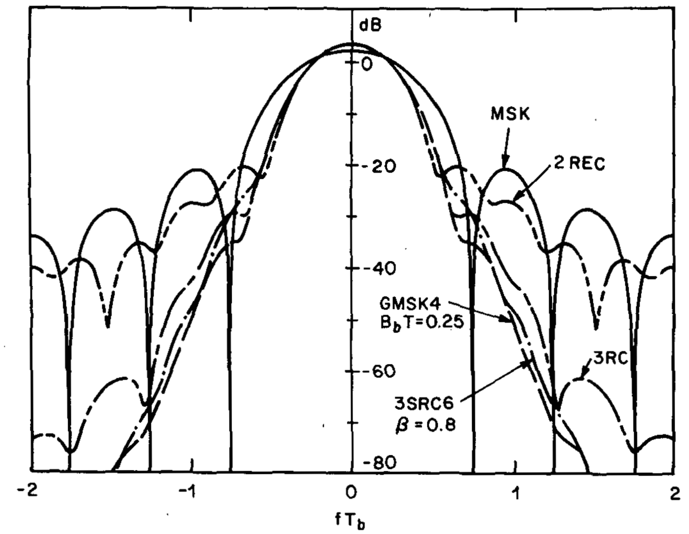

Fig. 3. Average power spectrum for some binary CPM schemes with h = 1/2, see Table 1.

lihood sequence detection (Viterbi detection) is the minimum Euclidean distance between all possible pairs of signals

$$D^2_{min} = d^2_{min} \cdot 2E_b = min\left\{2E_b\, log_2(M)\, \frac{1}{T}\int_0^{NT} [1-cos[\phi(t,\underline{\alpha}) - \phi(t,\underline{\beta})]]dt\right\} \quad (4)$$

where E_b is the signal energy per bit given by $E_b log_2(M) = E$ and where NT is the receiver observation interval length. When N is sufficiently large, the free distance, (the largest obtainable minimum distance) is reached.

For ideal coherent transmission over an additive white Gaussian noise channel, the bit error probability for high signal-to-noise ratios, E_b/N_0, is, to a good approximation:

$$P_b \approx Ce^{-d^2_{min}\cdot\frac{E_b}{N_0}}. \quad (5)$$

where C is a constant.

Efficient algorithms exist for computing the minimum distance for different $g(t)$, L, h and M [5,39]. Figure 5 shows $d^2_B \geq d^2_{min}$ as a function of h for binary LRC. d^2_B equals the free distance $d^2_f = d^2_{min}$ for almost all h-values in Fig. 5 [5]. We note from this figure, that the distance grows with L at least for large h. The vertical axis of Fig. 5 also has a dB scale which gives the gain in E_b/N_0 relative to MSK (d^2_{min}=2) for low P_b's (see (5)).

The comparisons in Fig. 5 are somewhat artificial, since the bandwidth also changes with L and h. Another comparison of CPM schemes is given in the scatter plot in Fig. 6 where each point represent a CPM scheme with its 99 percent power bandwidth $2BT_b$ ($T_b = T/log_2(M)$) and the signal-to-noise ratio difference relative to MSK (coding/modulation gain) in dB for high E_b/N_0. Thus, schemes on the same vertical line (for example, through the MSK point) have the same bandwidth at equal data rates. Schemes on the same horizontal line have the same error probability for high signal-to-noise ratios. Thus, it is evident that larger L and larger M yield more efficient systems. Not surprisingly, the system complexity increases in the same direction. We have marked some binary h = 1/2 schemes, some binary 3RC schemes and some quaternary 3RC schemes on Fig. 6. The h = 2/3 and

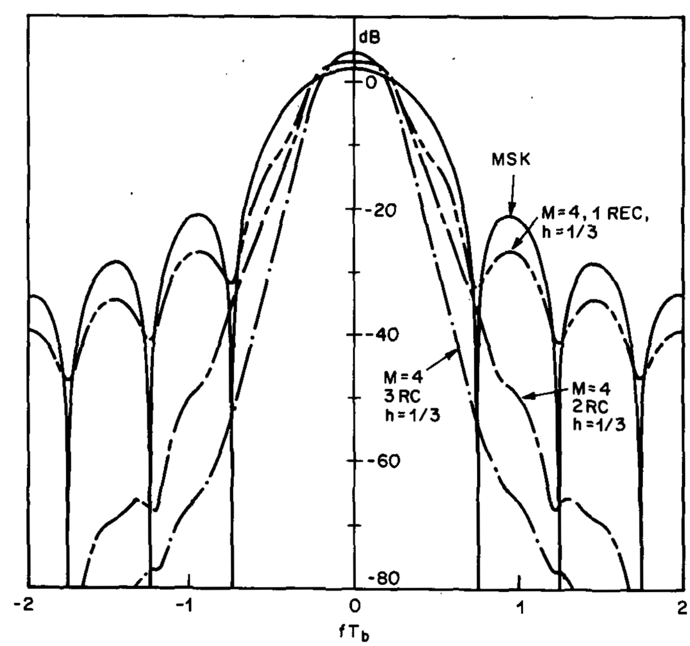

Fig. 4. Average power spectra for MSK (M=2, 1REC, h=1/2) and the M = 4, h = 1/3 schemes with 1REC, 2RC and 3RC pulses.

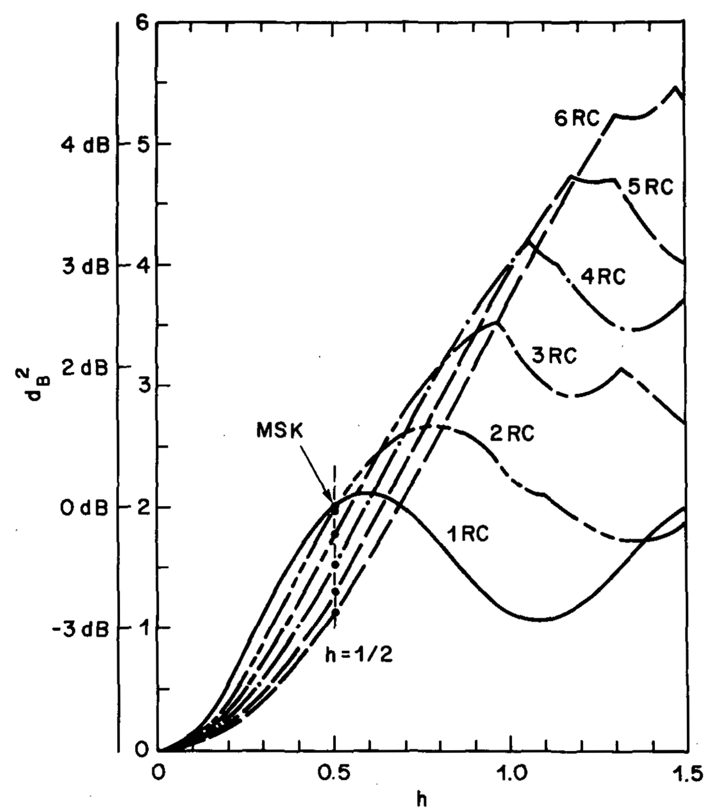

Fig. 5. Upper bound d^2_B on the distance for the binary CPM schemes 1RC, 2RC, ···, 6RC.

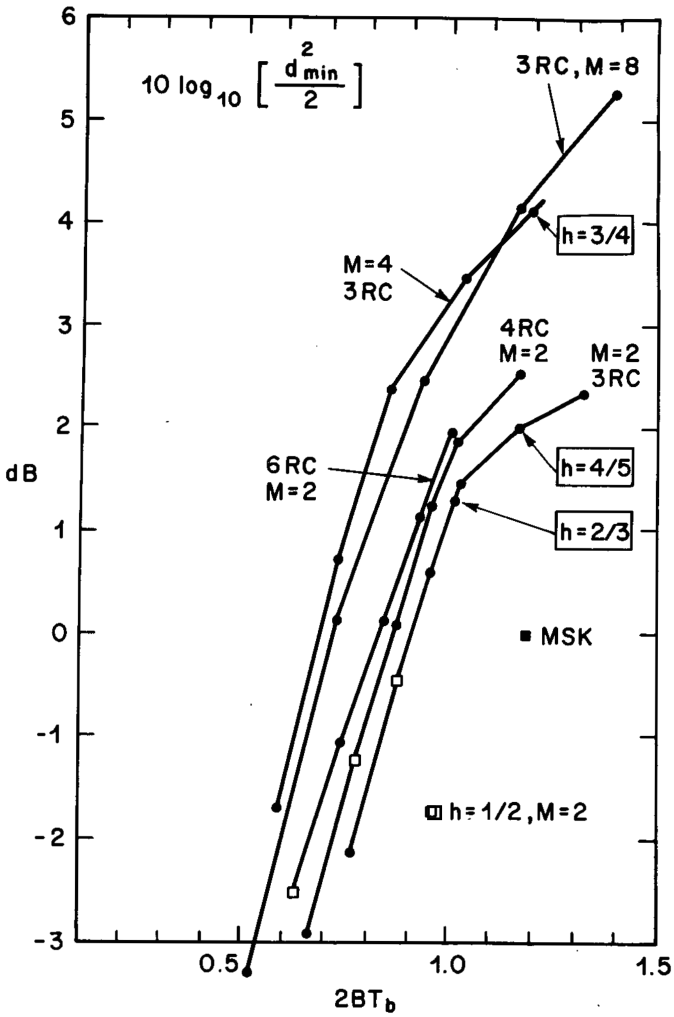

Fig. 6. Power-bandwidth tradeoff for CPM schemes using RC-pulses. The bandwidth is the 99% power in band definition. Note that the specific schemes are plotted as points and these are interconnected with straight lines.

4/5 binary 3RC schemes correspond to the phase cylinders in Fig. 2a and 2c. The number of states is 12 and 20 respectively.

Transmitters

A conceptual general transmitter structure based on (1) is shown in Fig. 7. This structure is however not easily converted into hardware for coherent systems. The reason is that an *exact* relation between the symbol rate and the modulation index is required and this requires control circuitry. On the other hand an advantage with this structure is that it can be used both for analog and digital inputs.

The most general and straight forward way of implementing a robust CPM-transmitter is to use stored look-up tables. This is seen by rewriting the normalized CPM waveform $S_0(t,\underline{\alpha}_n) = S(t,\underline{\alpha}_n)/\sqrt{\frac{2E}{T}}$ as:

$$S_0(t,\underline{\alpha}_n) = I(t)\cos(2\pi f_0 t) - Q(t)\sin(2\pi f_0 t) \qquad (6)$$

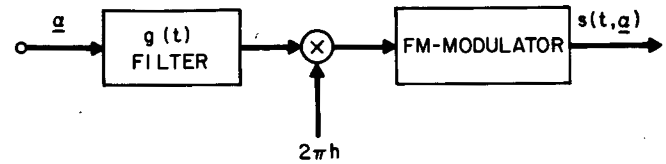

Fig. 7. Conceptual modulator for CPM directly based on Equation (1).

where $I(t) = \cos(\theta(t,\underline{\alpha}_n) + \theta_n)$ and $Q(t) = \sin(\theta(t,\underline{\alpha}_n) + \theta_n)$. The subscript n on $\underline{\alpha}_n$ indicates that we are considering the data symbol α_n and sufficiently many of the previous symbols. Figure 8 shows a transmitter based on Equation (6) where the two read only memories contain sampled and quantized versions of I(t) and Q(t) for each data symbol α_n, correlative state vector (the L−1 previous data symbols) and phase state value. The address field for the ROM is roughly $L \log_2(M) + [\log_2(p)] + 1$ bits and the ROM size is $p \cdot M^L \cdot m \cdot m_q$ bits where m is the number of samples per symbol time and m_q is the number of bits per quantized sample.

The transmitter in Fig. 8 also contains a small sequential machine with a phase state look up table for calculating the next phase state, given the previous one and the incoming data symbol. The transmitter also contains two D/A converters.

For example for binary 3RC with $h = 2/3$, the ROM-address length is five bits and the ROM size is 1024 bits. For a wide range of CPM parameters, the ROM-size does not seem to be a problem. Speed and accuracy of transmitters of this type is discussed in [5,40]. Similar alternative transmitter structures are possible, where the size of the look up tables can be further reduced.

Several special modulator structures have been devised for MSK. Because MSK is a quadrature-multiplexed modulation scheme, it can be optimally detected by coherently demodulating its in-phase and quadrature components in parallel. Clearly, the quadrature chan-

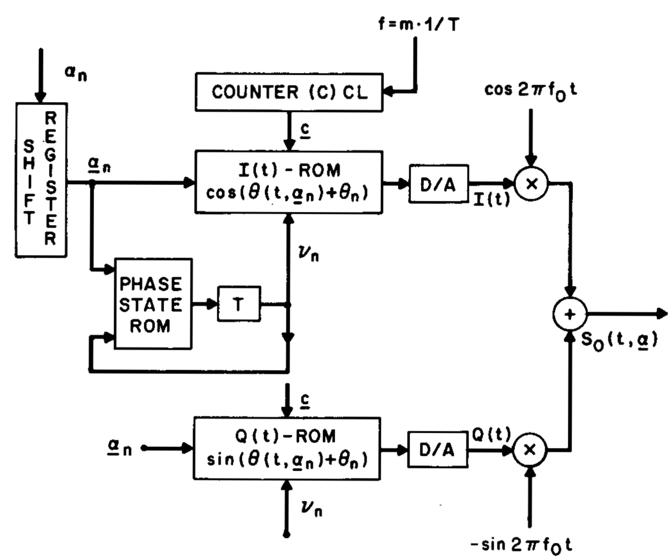

Fig. 8. General CPM transmitter based on the look-up table principle.

nels of the modulator and demodulator must be time synchronized, amplitude balanced, and in phase quadrature, to avoid overall system degradation. The serial method is an alternative approach to parallel modulation and demodulation of MSK which avoids some of these problems [3,41,42].

The serial modulation of MSK is somewhat more subtle to grasp than the parallel method. A serial modulator structure for MSK is illustrated in Fig. 9. It consists of a pure BPSK modulator with carrier frequency of $f_0-1/4T$ Hz, and a bandpass conversion filter with impulse response:

$$h(t) = \begin{cases} \frac{1}{T} \sin 2\pi\,(f_0+1/4T)t & 0 \leq t \leq T \\ 0 & otherwise \end{cases} \qquad (7)$$

which corresponds to a $(\sin x)/x$-shaped transfer function.

It is not completely clear how this idea might be generalized to a serial partial response transmitter. MSK is very special, since it corresponds to linear modulation in the quadrature arms [2-5]. This is not the case for partial response CPM with for instance 3RC, $M=2$, $h=1/2$. A good approximation of a serially generated 3RC CPM signal can be obtained by filtering and hardlimiting the serial MSK signal [16]. Serial MSK-type detection can, however, be applied to partial response CPM. This will be discussed in the next section.

Receivers

Receivers for coherent CPM have been and are still an active area of research [43-57]. For a general CPM scheme with a rational modulation index h and a pulse of finite length L it is straightforward to show that ideal optimum coherent detection can be performed by means of the Viterbi algorithm. The state and trellis description discussed earlier is used. The metric is calculated in a bank of linear filters which are sampled every symbol interval. Figure 10 shows the conceptual diagram for this general receiver. The path memory in the trellis processor cause a delay of N_T symbol intervals. This path memory is related to the growth of the minimum distance with the observation interval length. N_T should be chosen sufficiently large so that the free distance d_f is obtained between all paths.

Although there are no theoretical problems in constructing a receiver based on the principles in Fig. 10 there are several practical ones. As with all Viterbi detectors, the complexity grows exponentially with signal

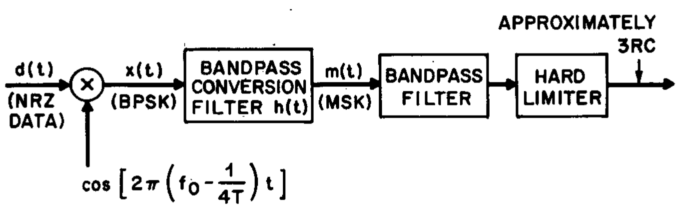

Fig. 9. Implementation of a serial MSK modulator utilizing a bandpass conversion filter. By proper bandpass filtering and hardlimiting an appropriate constant amplitude 3RC signal can be generated.

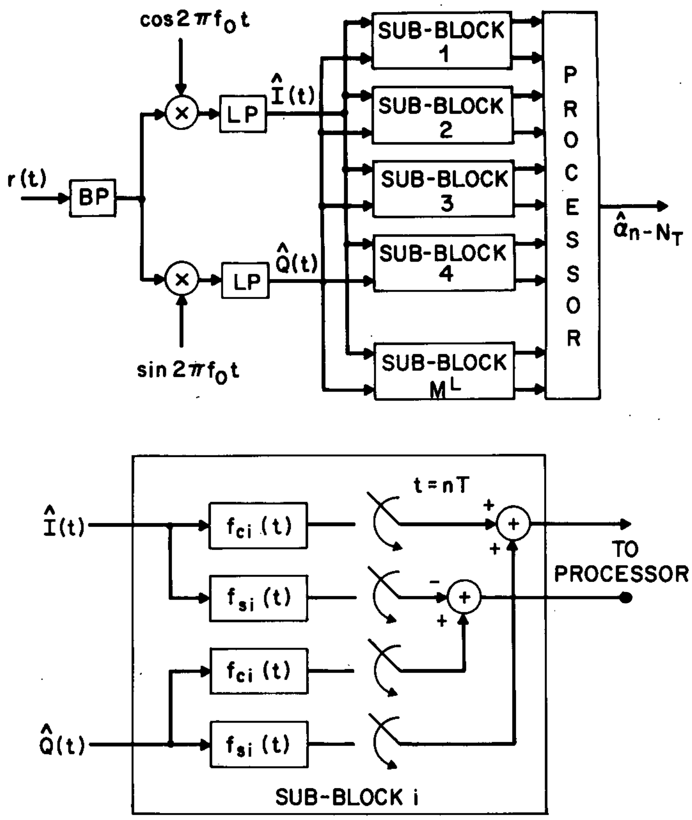

Fig. 10. A general Viterbi receiver structure for CPM. There are 4M^L linear filters.

memory. The limiting factors are the number of states $S = pM^{L-1}$ and the number of filters $F = 2M^L$ for calculating the metrics.

For many cases with long smoothing pulses, the optimum receiver can be approximated by a receiver based on a shorter pulse shape $g_R(t)$ of length $L_R < L$. Thus the complexity of the suboptimum receiver is reduced by a factor of M^{L-L_R} both for the number of states and the number of filters in the filter bank. The receiver pulse shape can be optimized for large signal-to-noise ratios for a given transmitter pulse shape $g(t)$ and a modulation index h. It is shown in [43] that the loss in error probability performance is very small when for example a binary 4RC scheme is received in a 2REC receiver or even smaller with an optimized receiver pulse $g_R(t)$ of length $L_R = 2$. The receiver phase tree (solid) and the transmitter phase tree are shown in Fig. 11 for this case for $h = 1/2$. Alternative general methods for reduced complexity receivers are given in [49] and [43].

As an example of complexity, consider the $h = 3/4$, $M = 4$, 3RC scheme marked in Fig. 6. This scheme has $p = 8$ phase states. However only four are used at each symbol interval [15]. For modulation index $h = 2k/p$ where p is even, $p_s = p/2$ states are used every symbol interval. This is obvious from Fig. 2b for $h = 1/2$, $M = 2$, 3RC. Thus, the number of states is $p_s M^{L-1} = 64$ and the number of filters is 128. These numbers can be reduced by a factor of 4 at a very slight performance loss by using the approximate receiver indicated by Fig. 11. This makes the number of states equal to 16 and the number of filters 32.

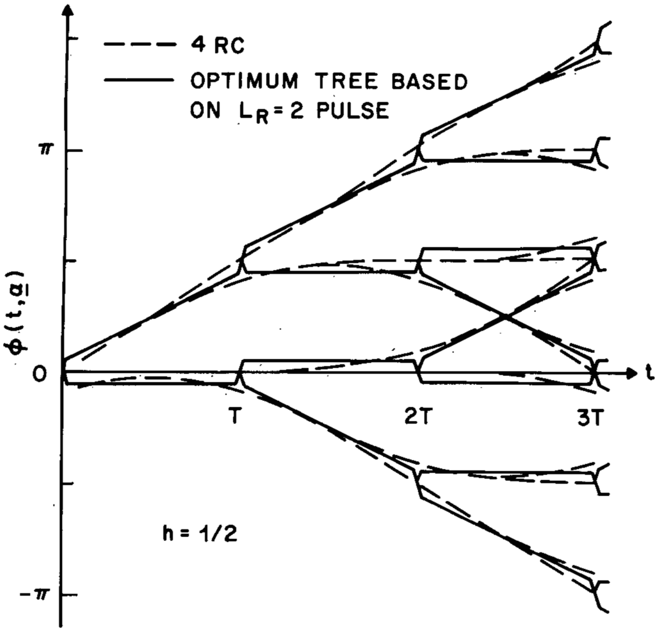

Fig. 11. Phase tree for 4RC compared to the phase tree for the optimum receiver based on a pulse of length $L_R = 2$.

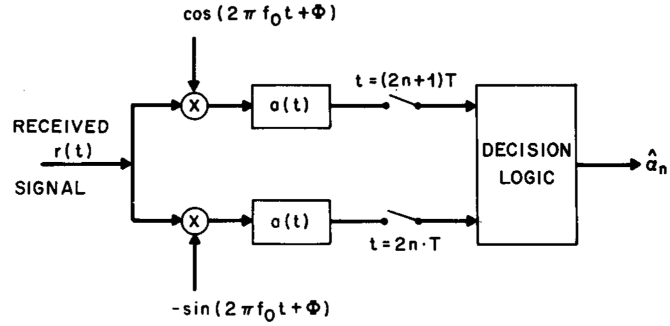

Fig. 12. Receiver structure for a parallel MSK-type receiver for h=*1/2 CPM.*

For some cases of CPM it is not necessary to use the Viterbi detector. Recently, much work has been devoted to the MSK-type receiver [5]. This receiver has only two filters and just a small amount of processing. The receiver makes single symbol decisions. This simplified receiver is of course in general suboptimum, but it works well for binary modulations with modulation index $h = 1/2$. Various ideas for selecting the receiver filters are analyzed in the early papers on this subject and an optimum filter is derived for various correlative FSK schemes (piecewise linear phase functions) in [56] and for smooth pulses in [45]. The performance for this type of receiver is almost equal to the optimum Viterbi receiver for schemes with a moderate degree of smoothing, that is, overlapping frequency pulses of length L up to 3 to 4 symbol intervals, like 3RC, 4RC, TFM and some GMSK schemes.

The MSK-type receiver is based on the structure of the parallel MSK receiver given in Fig. 12. Binary CPM-signals with modulation index 1/2 have attractive properties that can be used to formulate the simple MSK-type receiver, and we will exploit those here. Besides assuming $h = 1/2$, we will assume perfect carrier recovery and symmetric frequency pulses $g(t)$. The decisions are made every 2T in alternate quadrature arms. "Decision Logic" in Fig. 12 also consists of differential decoding and time demultiplexing as explained for MSK in [2].

It is known that MSK (1REC, $h=1/2$) is in fact a linear modulation in two quadrature arms and that the linear receiver in Fig. 12 is an optimum receiver for this case. We can also employ a linear receiver for partial response schemes with $h = 1/2$, although it may not be possible to express these schemes as linear modulation in the quadrature arms, and the receiver may not be optimum. Receiver decisions in one of the quadrature arms will be made just from the $\cos[\phi(t,\underline{\alpha})]$ eye pattern. Figure 13 gives an example for 3RC. The other quadrature arm is based on the $\sin[\phi(t,\underline{\alpha})]$ eye pattern, which is identical to the $\cos[\phi(t,\underline{\alpha})]$ eye pattern but shifted by T in time.

Thus, for 3RC, the signal set in Fig. 13 is passed through the linear filter a(t) in Fig. 12 and a binary decision is made each 2T sec. In the other quadrature arm, the same signal set is passed through the same receiver filter except it has a time offset T. There are algorithms for optimizing the receiver filter $a(t)$ for different filter length $N_F T$, transmitter pulse $g(t)$ and different channel signal-to-noise ratios, E_b/N_0. The optimum receiver filter minimizes the average symbol error probability [42,53]. Figure 9 shows the receiver filter $a(t)$ of length 9T for two signal-to-noise ratios. The AMF aver-

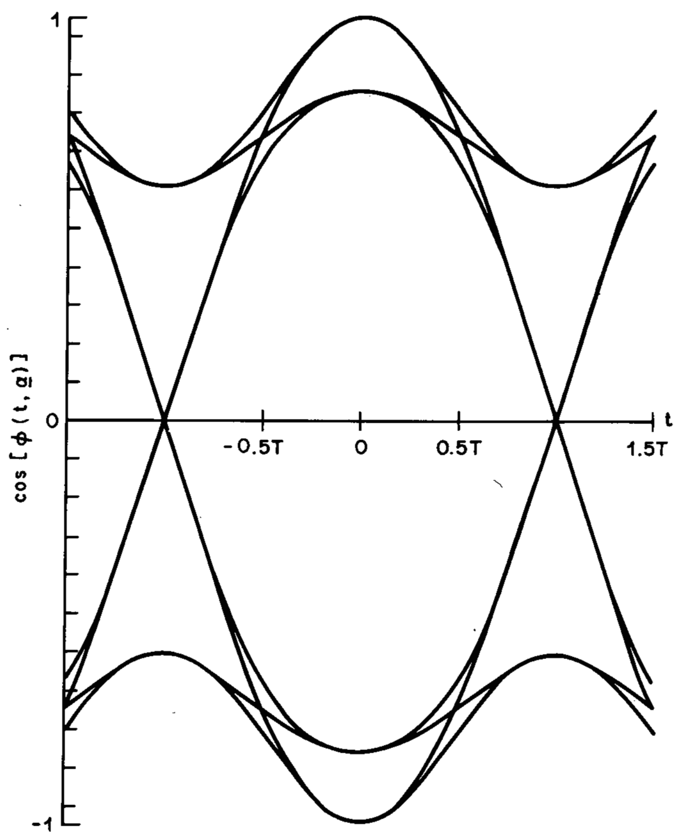

Fig. 13. Eye pattern for parallel MSK-type receiver for 3RC, h = *1/2.*

age matched filter is optimum in the limit for low SNR's and the asymptotic filter is optimum for high SNR's. The asymptotic filter is nearly optimum for all signal-to-noise ratio's of interest in practice. Figure 15 shows the symbol error probability for some binary $h = 1/2$ CPM schemes with optimum receiver filter. Note the 3RC is almost as good as MSK while TFM and 4RC are about 1 dB worse in E_b/N_o at an error probability of 10^{-3}. The difference grows with SNR. Figure 15 shows the error probability before differential decoding. After differential decoding, the bit error probability for all the schemes is about twice of those in Fig. 15.

Thus, the parallel MSK-type receiver works well for schemes based on smooth frequency pulses with a main-lobe width of four symbols or less. One disadvantage of this type of receiver is that decisions must be made alternately in the quadrature arms, adding hardware complexity in a practical receiver. Recently another receiver of serial type has been investigated which avoids this disadvantage. It is also easier to implement and is less sensitive to phase errors. The serial receiver has been studied and its optimum receiver filters have been found for MSK and partial response CPM. The error performance has been analyzed and results for phase errors have been presented, [3,41-43].

The optimum serial receiver for MSK is shown in Figure 16; compare it to the parallel receiver structure in

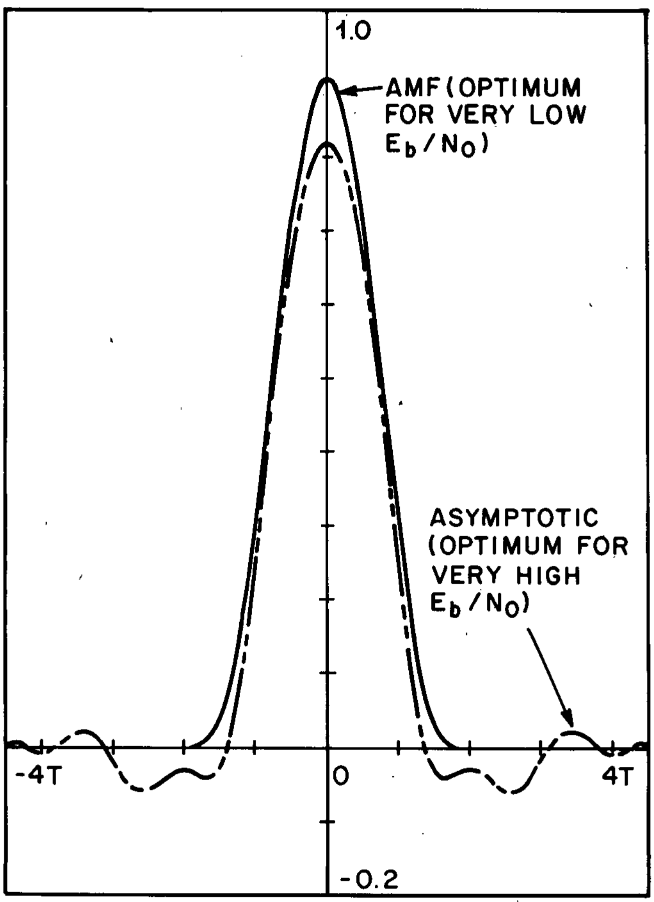

Fig. 14. The asymptotically optimum filter for 3RC with length $N_F = 9$.

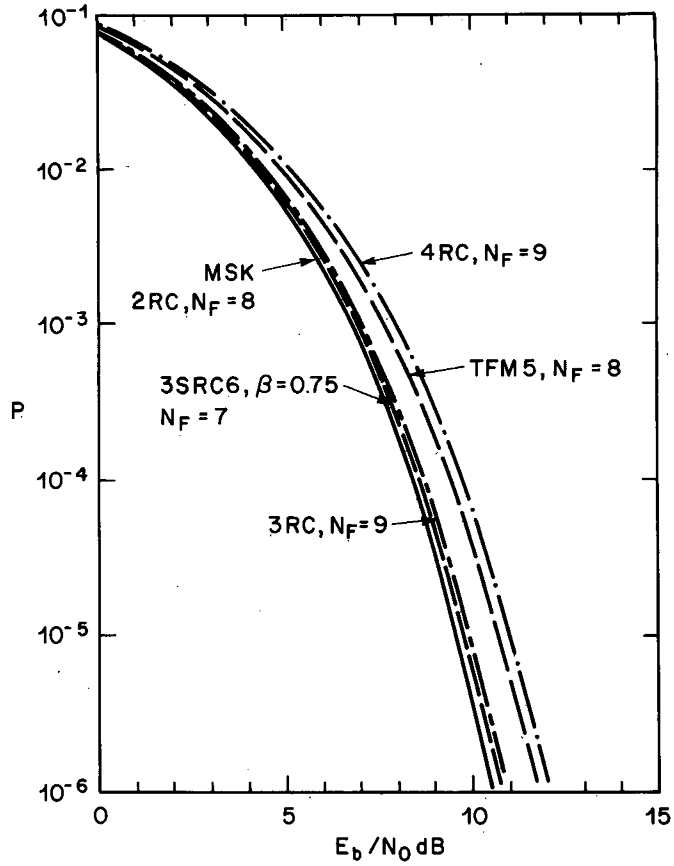

Fig. 15. The error probability for some binary, h = *1/2 CPM schemes. The MSK-type receivers use asymptotically optimum receiver filters for length* N_F.

Fig. 12. From [5], we have the filter $a(t)$ in the parallel receiver for MSK, that is:

$$a(t) = \begin{cases} \cos(\frac{\pi t}{2T}) & ; |t| < T \\ 0 & ; \text{otherwise.} \end{cases} \quad (8)$$

We have introduced a phase error Φ in the figure in anticipation of the phase recovery analysis to follow. Perfect synchronization corresponds to $\Phi = 0$. The serial receiver for MSK uses two filters defined by:

$$h_1(t) = \begin{cases} \cos^2 \frac{\pi t}{2T} & ; |t| \le T \\ 0 & ; \text{otherwise.} \end{cases}$$
$$h_2(t) = \begin{cases} -\frac{1}{2} \sin \frac{\pi t}{T} & ; |t| \le T \\ 0 & ; \text{otherwise.} \end{cases} \quad (9)$$

The difference compared to the parallel receiver is the demodulation frequency (IF). In the serial receiver the signal is demodulated with the frequency $f_l = f_0 - \frac{1}{4T}$. This frequency corresponds to the transmitted frequency for an all "-1" transmitted data sequence. Thus the signals in the quadrature arms of the serial receiver are $\cos[\phi_1(t,\underline{\alpha})]$ and $\sin[\phi_1(t,\underline{\alpha})]$ where $\phi_1(t,\underline{\alpha}) = \phi(t,\underline{\alpha}) + \frac{\pi t}{2T}$.

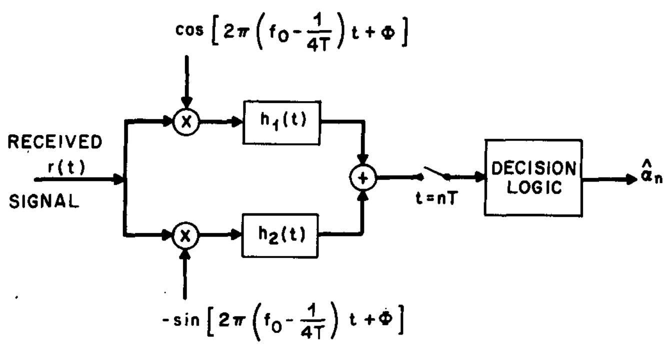

Fig. 16. Receiver structure for a serial MSK-type receiver for binary h $= 1/2$ *CPM.*

These are equivalent to the unfiltered quadrature signals for a parallel receiver when the transmitted constant amplitude signal has the phase function $\phi_1(t,\underline{\alpha})$. Note that a transmitted all "-1" sequence now corresponds to a zero phsae trajectory. These quadrature signals are then filtered and added. The advantage of the serial eye is the open eye at every symbol interval, which means that all the symbols can be detected from this eye. The eye opening is the same as the parallel eye. Therefore the ideal performance for these two receivers is the same.

For the general $h = 1/2$ binary CPM scheme, the filters $h_1(t)$ and $h_2(t)$ in (9) can be easily derived from the filter $a(t)$ for the parallel receiver. The relation is given by:

$$\begin{cases} h_1(t) = a(t) \cdot cos \dfrac{\pi t}{2T} \\ h_2(t) = -a(t) \cdot sin \dfrac{\pi t}{2T} . \end{cases} \qquad (10)$$

It is reasonable, therefore, that the serial receiver could also be used for partial response schemes, with the filters chosen according to (10) and $a(t)$ the corresponding filter for the parallel receiver.

It is shown in [43,46] that the performance with serial MSK-type receiver is equal to that of the parallel MSK-type receiver for all ideal optimum receivers. Serial and parallel MSK-type detection with phase errors and timing errors are compared for partial response CPM. It is concluded that the same relative behavior that is observed for MSK is also true for binary partial response CPM signals with $h = 1/2$. Thus serial detection is less sensitive to phase errors than parallel detection while serial is more sensitive to timing errors. Often in practice, the phase error sensitivity is more important than the timing error sensitivity.

As an example of the performance of the serial MSK-type receiver we have calculated the serial and parallel detection eye with the suboptimum $a(t)$ given by (8) for 3RC with a phase error of $\Phi = -5°$, see Fig. 17. Note that the detection eye is the same for $t = T$ while it is better for serial detection at time offset $T + \Delta T_0$ where, $\Delta T_0 = -T/18$.

Figure 18 shows the relative degradation in SNR for an error probability of 10^{-6} for parallel MSK, serial MSK and 3RC with two different receiver filters for serial MSK-type and parallel MSK-type receivers. Note that

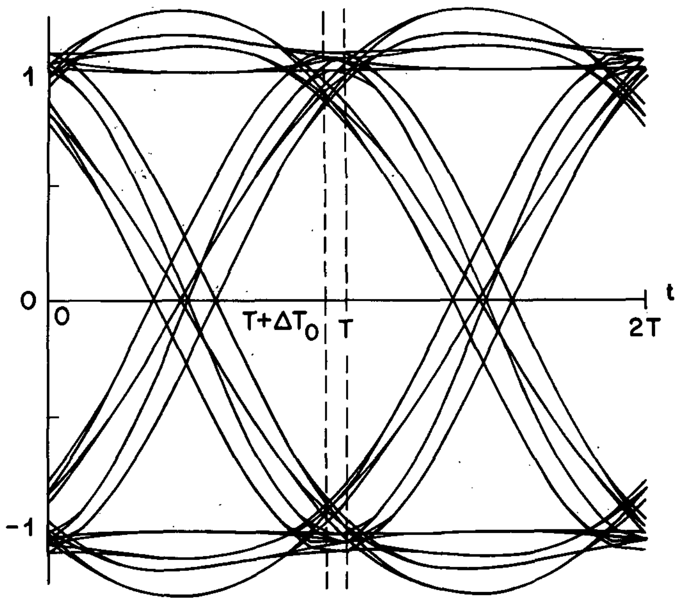

Fig. 17a. Decision eye for a serial MSK-type receiver for 3RC with phase error −5°.

the serial MSK-type receivers are consistently superior to the parallel receivers. It is obvious from Fig. 17, that the serial receiver is more sensitive to timing errors than the parallel receiver.

Discussion

In previous sections, we have presented a brief overview of some properties of the CPM signals. We should however also comment on the problems that have not been addressed in this article. The most important one is perhaps synchronization. Generalizations of the methods in [4] and [58] can be applied to CPM, [5,59–63]. However carrier phase recovery and symbol time extraction

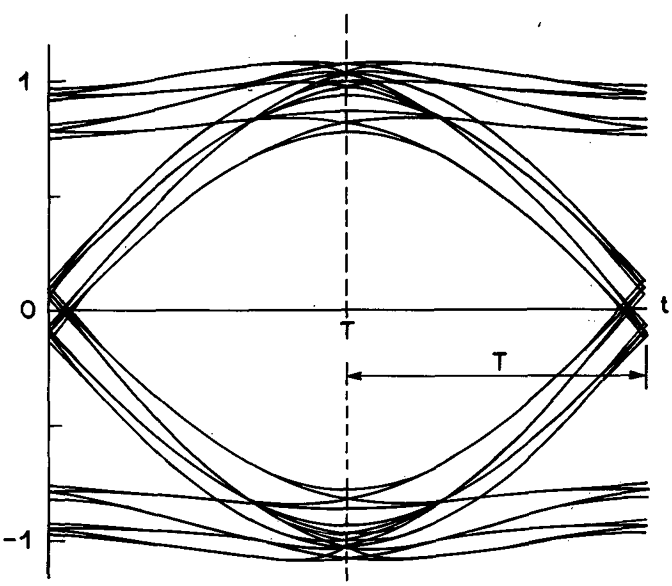

Fig. 17b. Decision eye for a parallel MSK-type receiver for 3RC with phase error −5°.

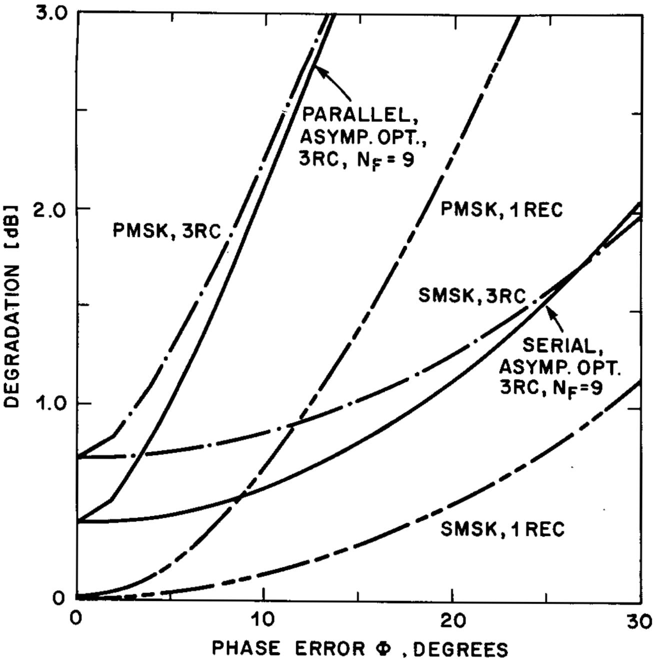

Fig. 18. Degradation in E_b/N_0 in dB relative to ideal MSK for bit error probability $P = 10^{-6}$ versus phase error Φ for 3RC and MSK-filter and asymptotically optimum filter for parallel and serial MSK-type detection. For comparison, PMSK and SMSK are also shown.

for general CPM requires further study. Some results are available on the effect of filtering and hard limiting of CPM and related problems, [5,64-71]. More work is also required in this area.

There are further generalizations of CPM which we have not described in detail. One such scheme is multi-*h* phase coding [24-26] where the modulation index is made time-varying, typically cyclically. A larger class of schemes is obtained by combining convolutional coding with CPM [5,72-80]. It has been found that four-level schemes with two different modulation indices are about 2 dB better than the four-level fixed *h* schemes at the same bandwidth. The same relative relationship holds for binary schemes. Further improvements are obtained with, for example, rate 2/3 convolutional coding properly combined with eight-level CPM. The price associated with these improvements is increased complexity.

Another important problem area is in non-coherent reception. The continuous phase and the partial response memory can also be exploited by the receiver in this case, [81-98]. The analysis typically gets considerably more complicated. Various types of differential detection and discriminator detection have been proposed and analyzed or simulated.

Alternative channels should also be studied. Most of the results are for a Gaussian channel and some for a Rayleigh fading channel.

More experimental work and hardware implementation is of course needed. Modems for mobile radio seem to be emerging [23,99-103].

Summary

We have given a brief overview over some recent results for constant amplitude modulation schemes with low spectral sidelobes and good detection properties. The relationship between important system parameters such as the number of symbol levels, the smoothing pulse shape and the modulation index was studied for optimum coherent reception. A robust general transmitter scheme was presented. Some optimum and suboptimum receiver structures were studied for ideal coherent transmission. We may conclude that the popular binary TFM and GMSK schemes can be somewhat improved by choosing the optimum receiver filter in the MSK-type receiver. Moreover, 3-5 dB more power efficient schemes with the same or similar average power spectral density can be obtained by changing the system parameters and using a Viterbi detector.

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