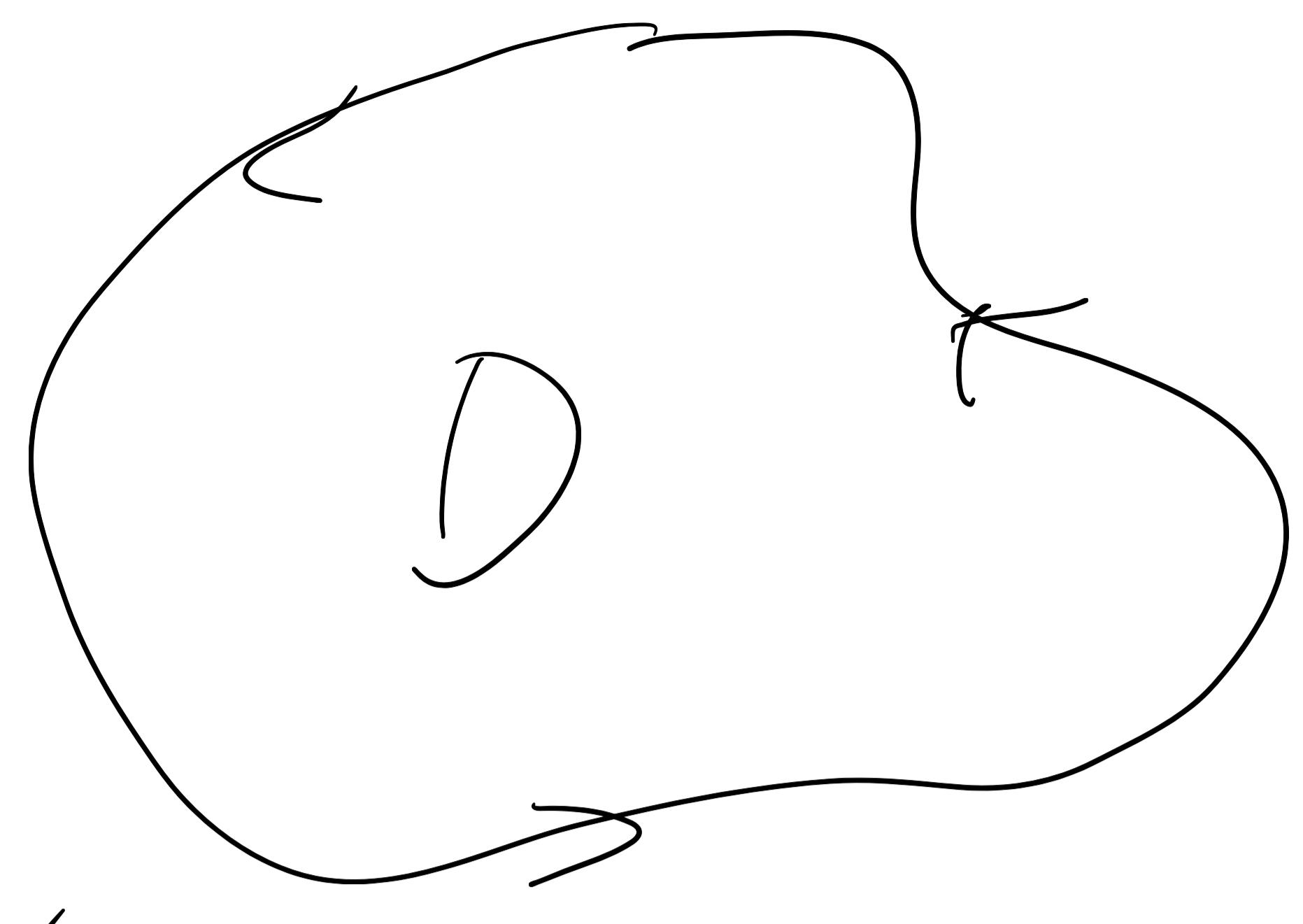
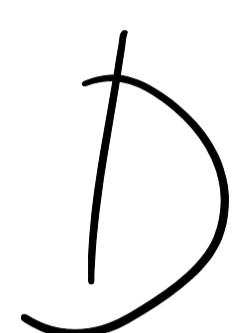


Ωεωρηγα Green

$$\int_{\partial D} P(x, y) dx + Q(x, y) dy$$



$$= \iint_D (Q_x - P_y) dx dy$$



$$\int_{\partial D} (P, Q) \cdot dS$$

$$\int_{\partial D} P \frac{\partial}{\partial x} + Q \frac{\partial}{\partial y} = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy$$

18. Let D be a region for which Green's theorem holds. Suppose f is harmonic; that is,

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$$

on D . Prove that

$$\int_{\partial D} \frac{\partial f}{\partial y} dx - \frac{\partial f}{\partial x} dy = 0.$$

$$\rightarrow = \iint_D (-\Delta f) dx dy = 0$$

Definition D and xw p/o.

To calculate the area of D we use

$$A(D) = \frac{1}{2} \int_{\partial D} x dy - y dx$$

$$= \int_{\partial D} x dy = - \int_{\partial D} y dx$$

A odd.

$$\int_{\partial D} x dy = \int_{\partial D} 0 \cdot dx + x dy$$

$$= \iint_D (\partial_x x - \partial_y 0) dx dy = \iint_D 1 dx dy$$

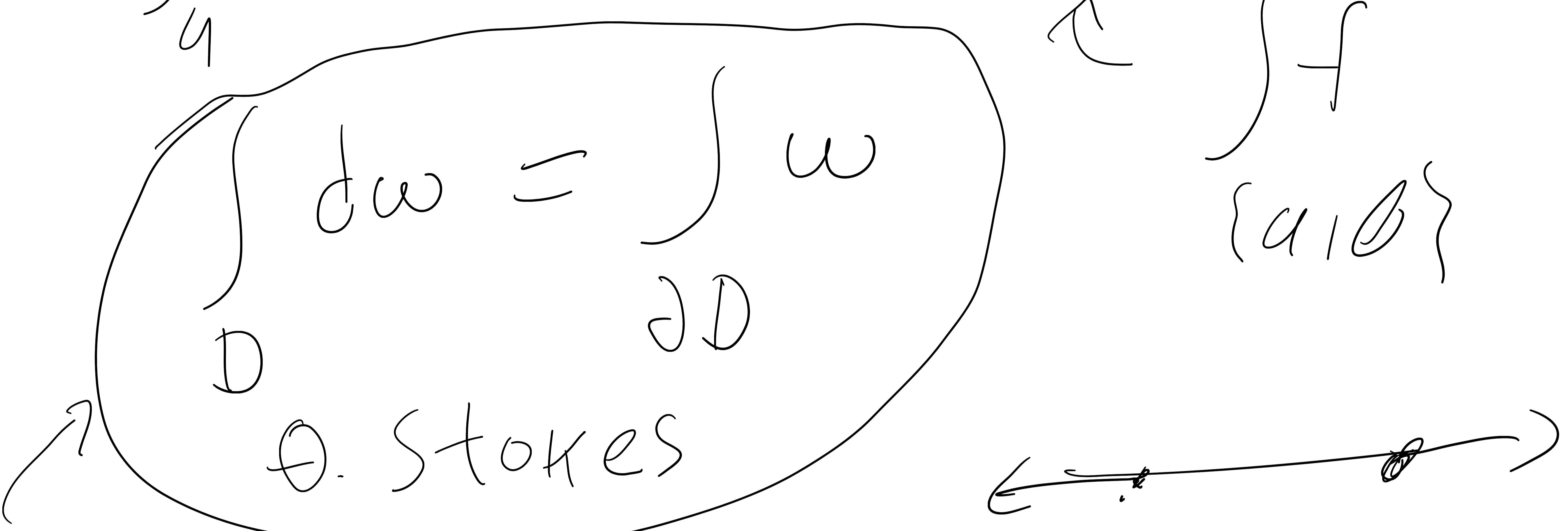
= ϵ p/u Jov ())

$$- \int_{\partial D} y dx = - \int_{\partial D} y dx + 0 dy =$$

$$= - \iint_D (0 - \partial_y Y) dx dy = \iint_D 1 dx dy$$

$$= \epsilon \mu \rho u \oint_{\partial D} (n)$$

$$\int_a^b f' dx = f(b) - f(a)$$

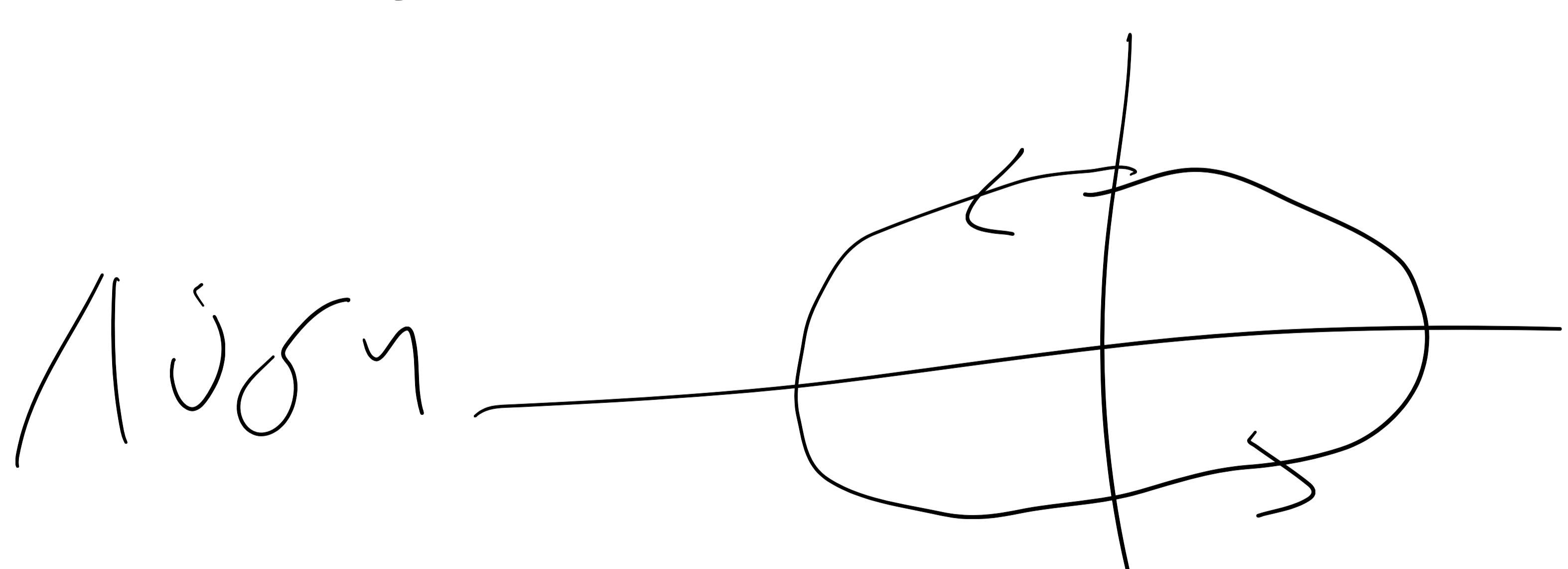


Marsden-Tromba §2.7 486. Example 14

Aşağıda \mathbf{H}_1 ile \mathbf{F} nin 10 eklemdeki

$$(1) \quad \text{Eşitliği} \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

\mathcal{C}_1 de



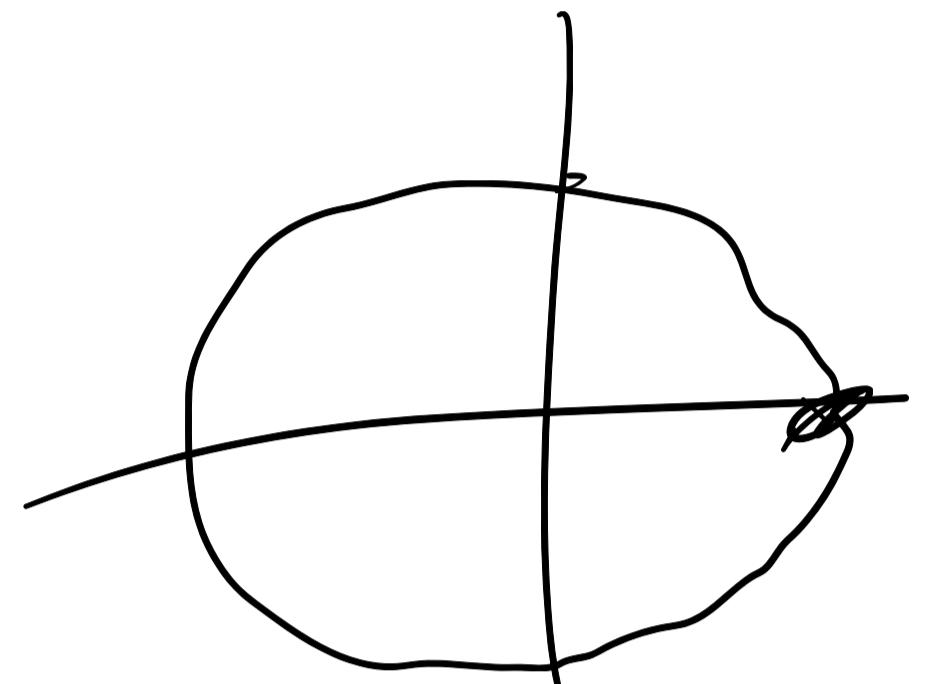
$$D = \left\{ (x, y) : \frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1 \right\}$$

$$2A(D) = \int_D -y \, dx + x \, dy$$

$$= \int_{\partial D} (-y, x) \cdot \hat{ds}$$

$x^2 + y^2 = 1$

$$\gamma: [0, 2\pi] \rightarrow \mathbb{R}^2$$



$$\gamma(t) = (\alpha \cos t, \beta \sin t)$$

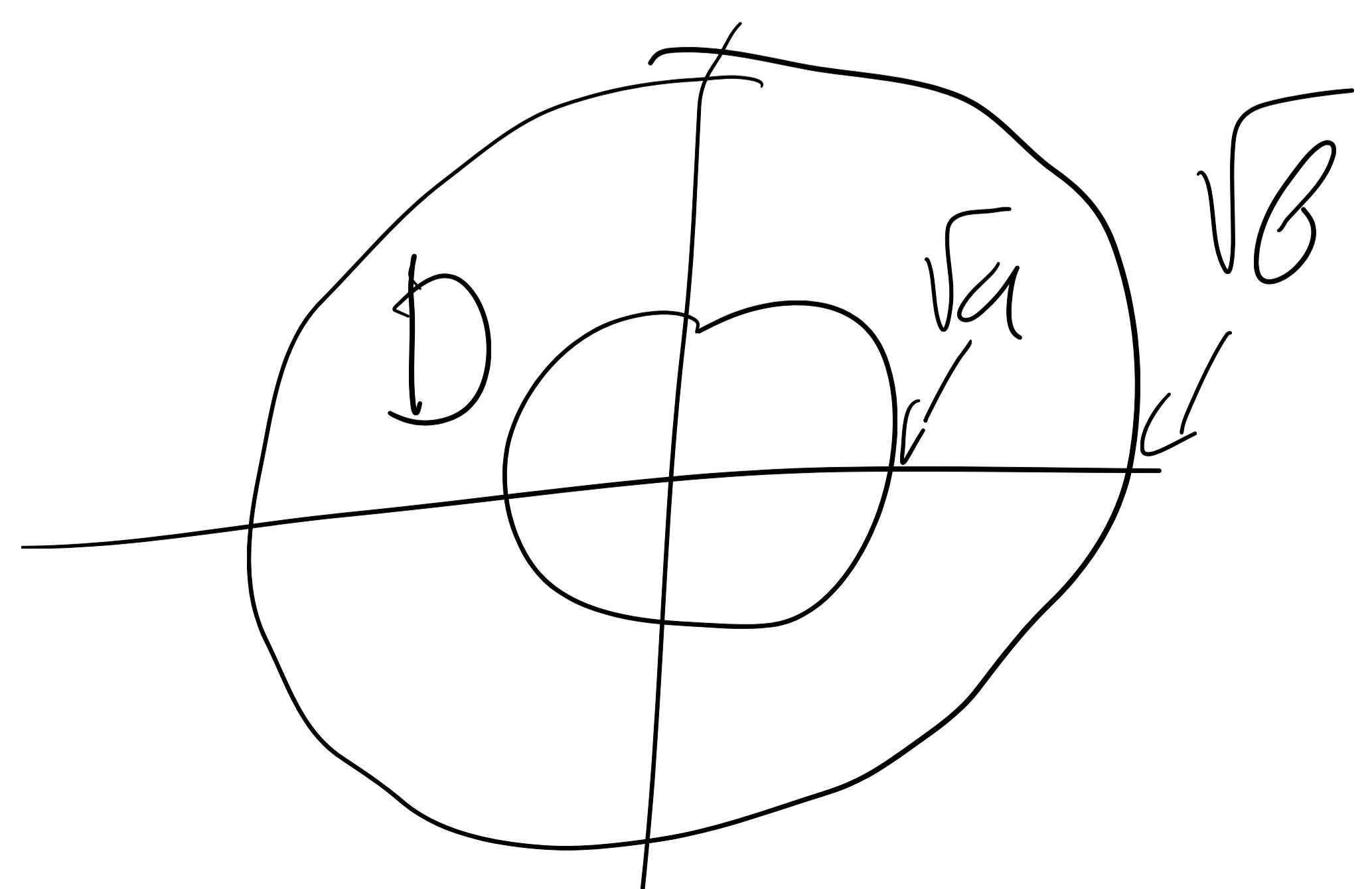
$$2A(D) = \int_0^{2\pi} (-\beta \sin t, \alpha \cos t) \cdot$$

$$(-\alpha \sin t, \beta \cos t) \, dt = \int_0^{2\pi} \alpha \beta \, dt$$

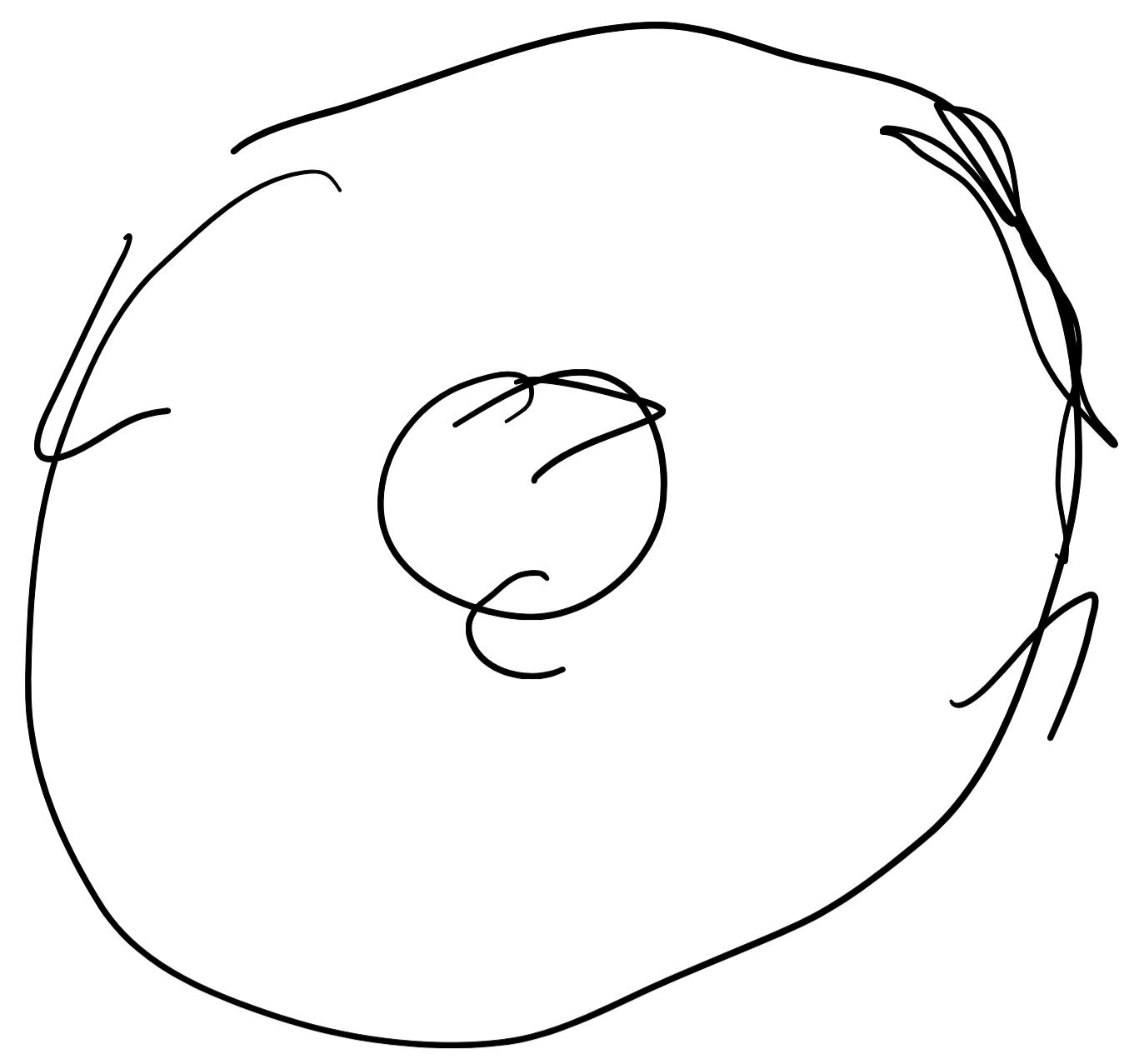
$$= 2\pi \alpha \beta \Rightarrow A(D) = \pi \alpha \beta$$

17. Verify Green's theorem for the integrand of Exercise 15 (that is, with $P = 2x^3 - y^3$ and $Q = x^3 + y^3$) and the annular region D described by $a \leq x^2 + y^2 \leq b$, with boundaries oriented as in Figure 8.1.5.

$$\int_{\partial D} P \, dx + Q \, dy \in$$



$$\left(\left(Q_x - P_y \right) \right) dx dy$$



$$D = \left(3x^t - (-3y^2) \right)$$

A handwritten mathematical expression involving a triple integral. The expression is:

$$= 3 \iiint_D (x^2 + y^2) dx dy$$

The region of integration is labeled D at the top left and bottom left. A curved arrow labeled t points from the right towards the integration limits.

$\text{Age} = \sqrt{\text{Age}} + \epsilon$

A hand-drawn diagram illustrating a process or transformation. On the left, there is a narrow, bell-shaped curve. To its right is a short vertical tick mark. Further to the right is a wider, more spread-out bell-shaped curve. An arrow points from the narrower curve towards the wider one. To the far right, a circle is divided vertically by a line, representing a split or a final state.

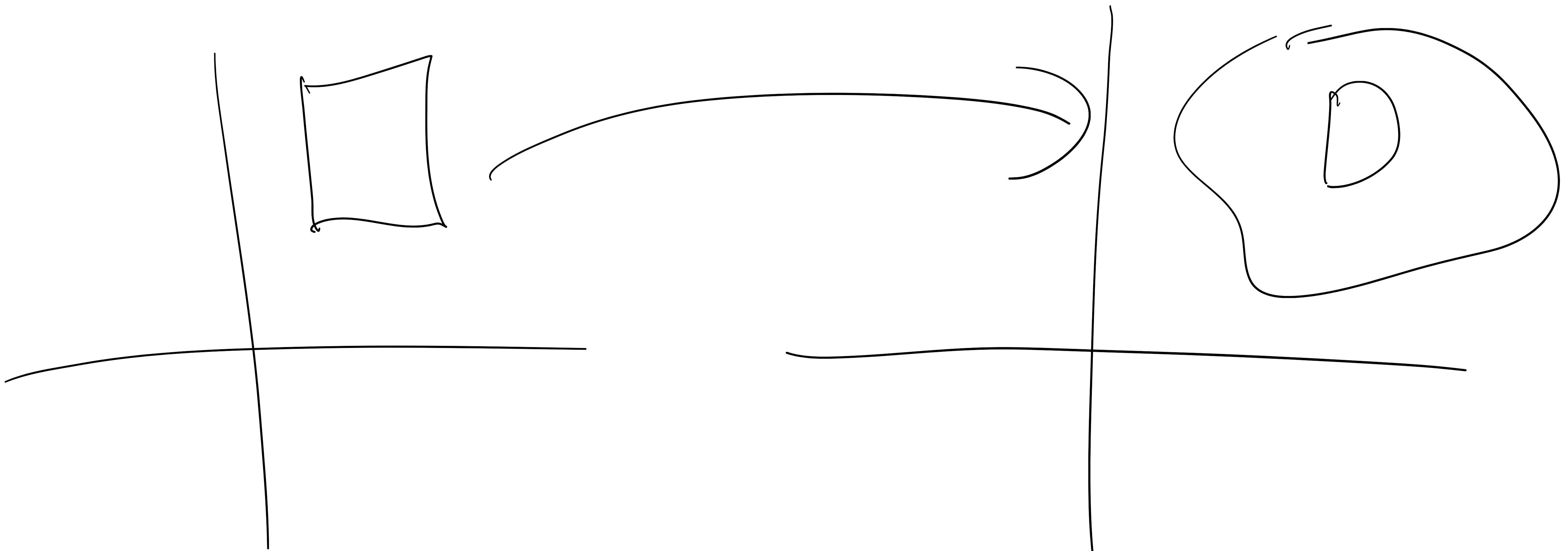
$$\pi(r, \theta) = (\sqrt{r} \cos \theta, \sqrt{r} \sin \theta)$$

$$\underline{T} = \int \int -[(x, y) \times d] = \int f(v, \theta, rs) dt$$

$$\int_A^D r^2 \sqrt{r} dr d\theta = \int_0^{2\pi} \int_0^{\sqrt{16}} r^3 dr d\theta$$

$$\leq 2P \frac{r^n}{4} \left| \frac{\sqrt{f}}{\sqrt{g}} \right| = \frac{n}{2} (\beta^2 - \alpha^2)$$

1. Ευθείες, τρίπλεξ, κατερικέ γνωμόνες
2. Κολλαγόνια με ειδικές προστάσεις
(κύριες αλυσίδες)
3. Ακρότητα
4. Διάδικτη πλάτη στοιχείων
(Fubini, Αλγορίθμοι)
5. Επικυρωτικά στοιχείων
6. Θ. Green $\int_T f$



$$\iint_D f = \iint_{T^{-1}(D)} f(T(u,v)) |J_T| du dv$$

Aufgabe 1 Na obere Umm 01

$$\partial_{xy}, \partial_x^2, \partial_y^2, T_{uv}$$

$$a) g(x,y) = x^2y + \sin(xy)$$

$$b) s(x,y) = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\tan: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \rightarrow \mathbb{R}$$

Aufgabe

$$a) \partial_x g = 2xy + \cos(xy) \cdot y$$

$$\partial_{xx} g = 2y - \sin(xy) \cdot y^2$$

$$\partial_{xy} g = 2x + \cos(xy)$$

$$-y \sin(xy) x$$

$$b) \partial_y s = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{1}{x} = \frac{x}{x^2+y^2}$$

$$\frac{\partial}{\partial y} S = x \left(-\frac{2y}{(x^2+y^2)^2} \right) \frac{d}{dt} (\tan^{-1}(t)) = \frac{-2xy}{(x^2+y^2)^2}$$

2. (2 μον.) Έστω $\varphi : \mathbb{R}^2 \rightarrow \mathbb{R}$ μία συνάρτηση με συνεχείς μερικές παραγώγους δεύτερης τάξης. Θέτουμε $f(x, y) := \varphi(x+y, x-y)$, $(x, y) \in \mathbb{R}^2$.

(α) Δείξτε ότι

$$\left[\frac{\partial f}{\partial x}(x, y) \right] \cdot \left[\frac{\partial f}{\partial y}(x, y) \right] = \left[\frac{\partial \varphi}{\partial x}(x+y, x-y) \right]^2 - \left[\frac{\partial \varphi}{\partial y}(x+y, x-y) \right]^2.$$

(β) Δείξτε ότι

$$\frac{\partial^2 f}{\partial x \partial y}(x, y) = \frac{\partial^2 \varphi}{\partial x^2}(x+y, x-y) - \frac{\partial^2 \varphi}{\partial y^2}(x+y, x-y).$$

Είσιντε u, v τα οριζόντια της φ . $\varphi(u, v)$

Θέτωμε $u(x, y) = x+y$

$v(x, y) = x-y$

a) $f(x, y) = \varphi(\underbrace{u(x, y)}, \underbrace{v(x, y)})$

$$\begin{aligned} \frac{\partial f}{\partial x}(x, y) &\equiv \frac{\partial \varphi}{\partial u} \left(\underbrace{u}_{\partial u}, \underbrace{v}_{\partial v} \right) \cdot \frac{\partial u}{\partial x}(x, y) + \frac{\partial \varphi}{\partial v} \left(\underbrace{u}_{\partial u}, \underbrace{v}_{\partial v} \right) \cdot \frac{\partial v}{\partial x}(x, y) \\ &= \frac{\partial \varphi}{\partial u} \left(\underbrace{u}_{\partial u}, \underbrace{v}_{\partial v} \right) + \frac{\partial \varphi}{\partial v} \left(\underbrace{u}_{\partial u}, \underbrace{v}_{\partial v} \right) \end{aligned}$$

$$\frac{\partial f}{\partial y}(x, y) = \frac{\partial \varphi}{\partial u} \frac{\partial u}{\partial y} + \frac{\partial \varphi}{\partial v} \frac{\partial v}{\partial y} = \varphi_u - \frac{\partial \varphi}{\partial v}$$

$$\frac{\partial f}{\partial x}(x, y) \cdot \frac{\partial f}{\partial y}(x, y) = \left(\frac{\partial \varphi}{\partial u}(x+y, x-y) \right)^2 - \left(\frac{\partial \varphi}{\partial v}(x+y, x-y) \right)^2$$

$$\frac{\partial f}{\partial x}(x, y) = \frac{\partial \varphi}{\partial u}(u(x, y), v(x, y)) + \frac{\partial \varphi}{\partial v}(u(x, y), v(x, y))$$

$$\frac{\partial^2 f}{\partial y \partial x}(x, y) = \frac{\partial^2 \varphi}{\partial u^2} \frac{\partial u}{\partial y} + \frac{\partial^2 \varphi}{\partial v \partial u} \frac{\partial v}{\partial y}$$

$$+ \frac{\partial^2 \varphi}{\partial v \partial u} \frac{\partial u}{\partial y} + \frac{\partial^2 \varphi}{\partial v^2} \frac{\partial v}{\partial y}$$

$$= \frac{\partial^2 \varphi}{\partial u^2} - \frac{\partial^2 \varphi}{\partial v \partial u} + \frac{\partial^2 \varphi}{\partial u \partial v} - \frac{\partial^2 \varphi}{\partial v^2}$$

$$= \frac{\partial^2 \varphi}{\partial u^2}(x+y, x-y) - \frac{\partial^2 \varphi}{\partial v^2}(x+y, x-y)$$

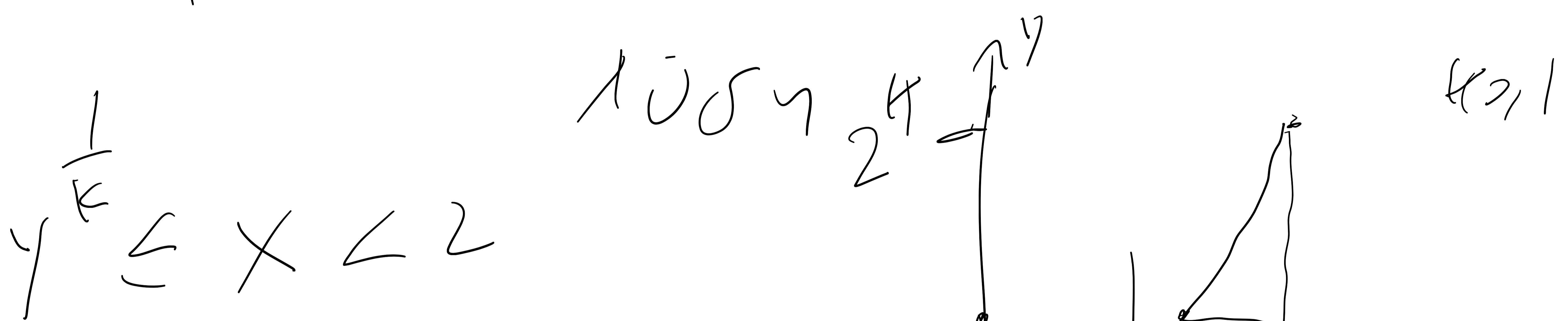
Άσκηση Α) Ηα υπολογιστεί το

$$I = \int_0^{2^k} \int_{\frac{y}{\sqrt{k}}}^2 e^{x^{k+1}} dx dy, \quad k \in \mathbb{R}$$

Β) Να λογάρισμος των χωρίου μέτρων

$$\text{από το σπασματικό } z = k(1 - \sqrt{x^2 + y^2})$$

Ηα φυλαξί των σύνδεσμων $z=0$, $z=k$,



$$D = \{(x, y) : 0 \leq x \leq 2 \\ 0 \leq y \leq x^k\}$$

$$I = \int_0^2 \int_0^{x^k} e^{x^{k+1}} dy dx \quad k \neq -1$$

$$= \int_0^2 e^{x^{k+1}} x^k dx = \frac{1}{k+1} \int_0^2 (e^{x^{k+1}})' dx$$

$$= \frac{1}{k+1} (e^{2^{k+1}} - e^0)$$

B) To calculate the volume of

$$D = \{(x, y, z) :$$

$$x^2 + y^2 \leq 1, 0 \leq z \leq k(1 - \sqrt{x^2 + y^2})\}$$

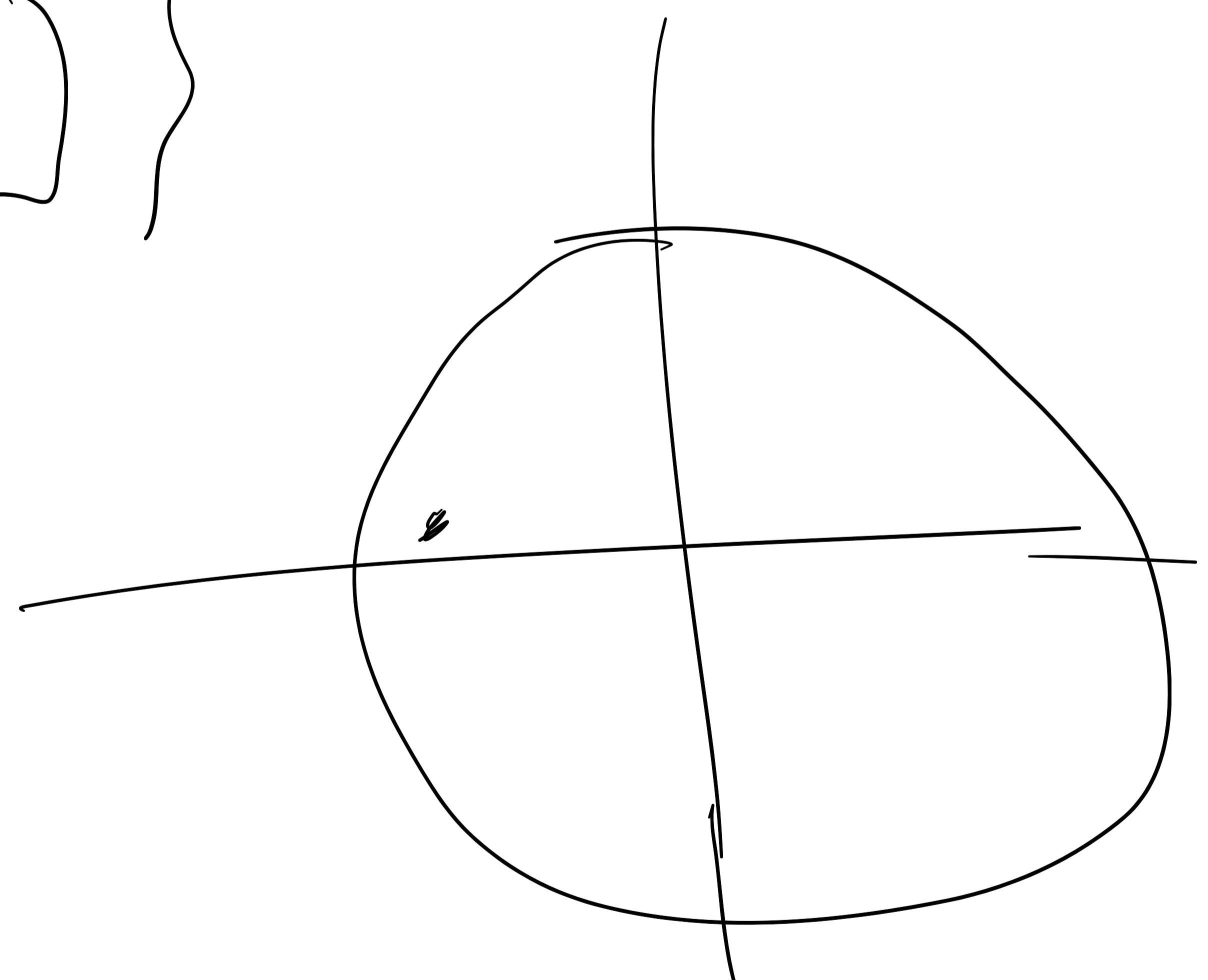
$$\text{volume}(D) = \iiint_D 1 dx dy dz$$

$$G = \{(r, \theta, \phi) : r \in [0, 1], \theta \in [0, 2\pi]\}$$

$$z \in [0, k(1-r)]\}$$

$$\text{volume}(D)$$

$$= \iiint_D 1 dx dy dz =$$



$$\iiint_G r dr d\theta dz = \int_0^{2\pi} \int_0^{\pi} \int_0^{k(1-r)} r dz d\theta dr$$

$$= \int_0^1 \int_0^{2\pi} r K(1-r) d\theta dr =$$

$$= 2\pi K \int_0^1 r(1-r) dr = 2\pi K \left(\frac{1}{2} - \frac{1}{3} \right) = \frac{\pi K}{3}$$

3. (25 Βαθμοί) Έστω $a > 0$. (α) Ζωγραφίστε στο επίπεδο τον κύκλο με εξίσωση $(x-a)^2 + y^2 = a^2$. Να βρεθεί η εξίσωσή του σε πολικές συντεταγμένες. [Δηλαδή γραφεί η εξίσωση του κύκλου με μεταβλητές τα r, θ που ορίζονται από τις $x = r \cos \theta, y = r \sin \theta$.]

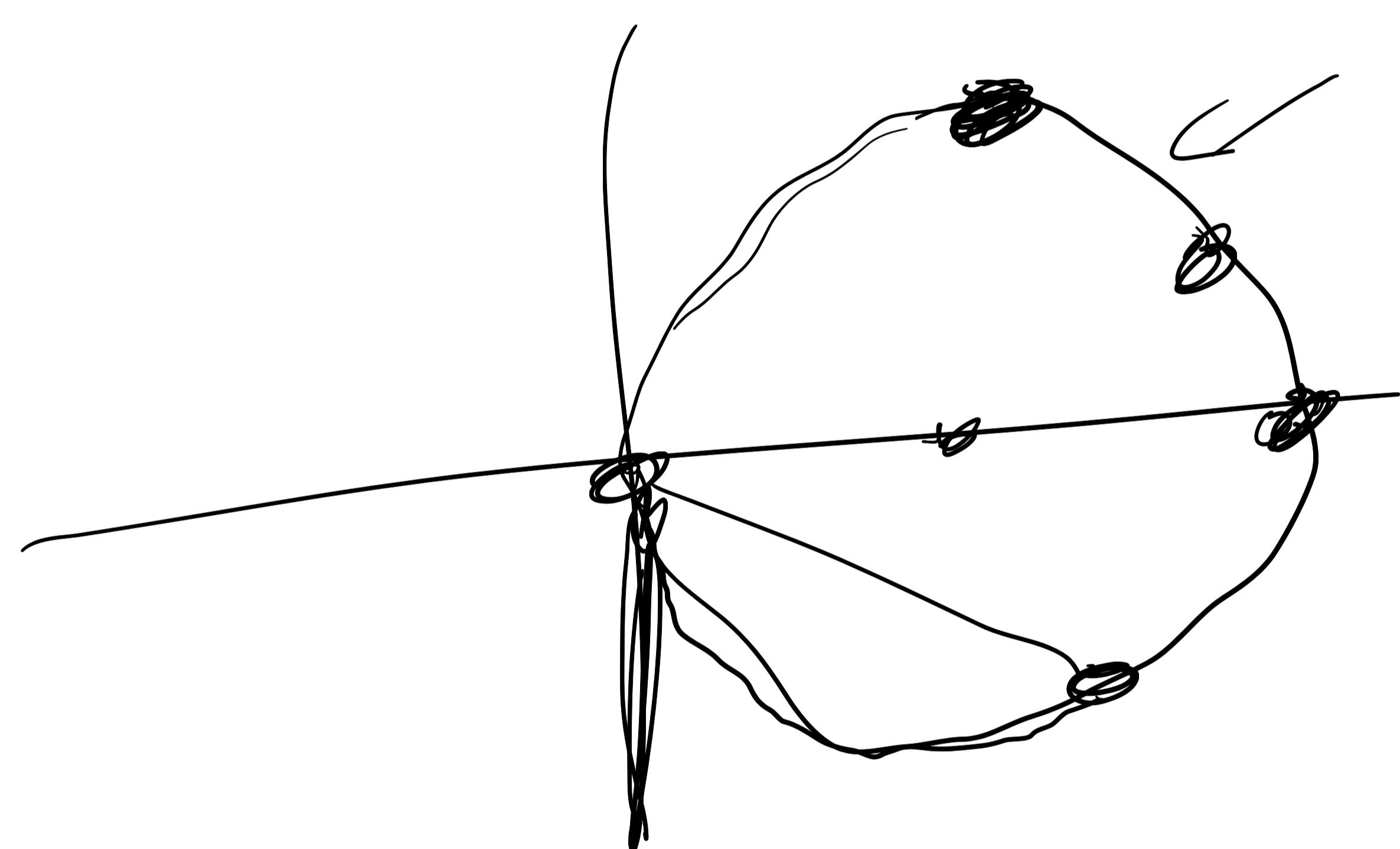
(β) Να υπολογιστεί με χρήση πολικών συντεταγμένων το ολοκλήρωμα

$$\iint_D \sqrt{x^2 + y^2} dx dy,$$

όπου $D := \{(x, y) \in \mathbb{R}^2 : (x-a)^2 + y^2 \leq a^2\}$.

$$x = r \cos \theta$$

$$y = r \sin \theta$$



~~$$r^2 \cos^2 \theta + \cancel{g^2}$$~~

$$-2a r \cos \theta + \cancel{r^2 \sin^2 \theta} = g^2$$

$$r^2 - 2a r \cos \theta = 0$$

$$r = 0$$

↙

$$r = 2a \cos \theta$$

↙ ↘

$$\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$$

$$r = 2a \cos \theta, \theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$$

0) $\iint_D \sqrt{x^2 + y^2} dx dy$

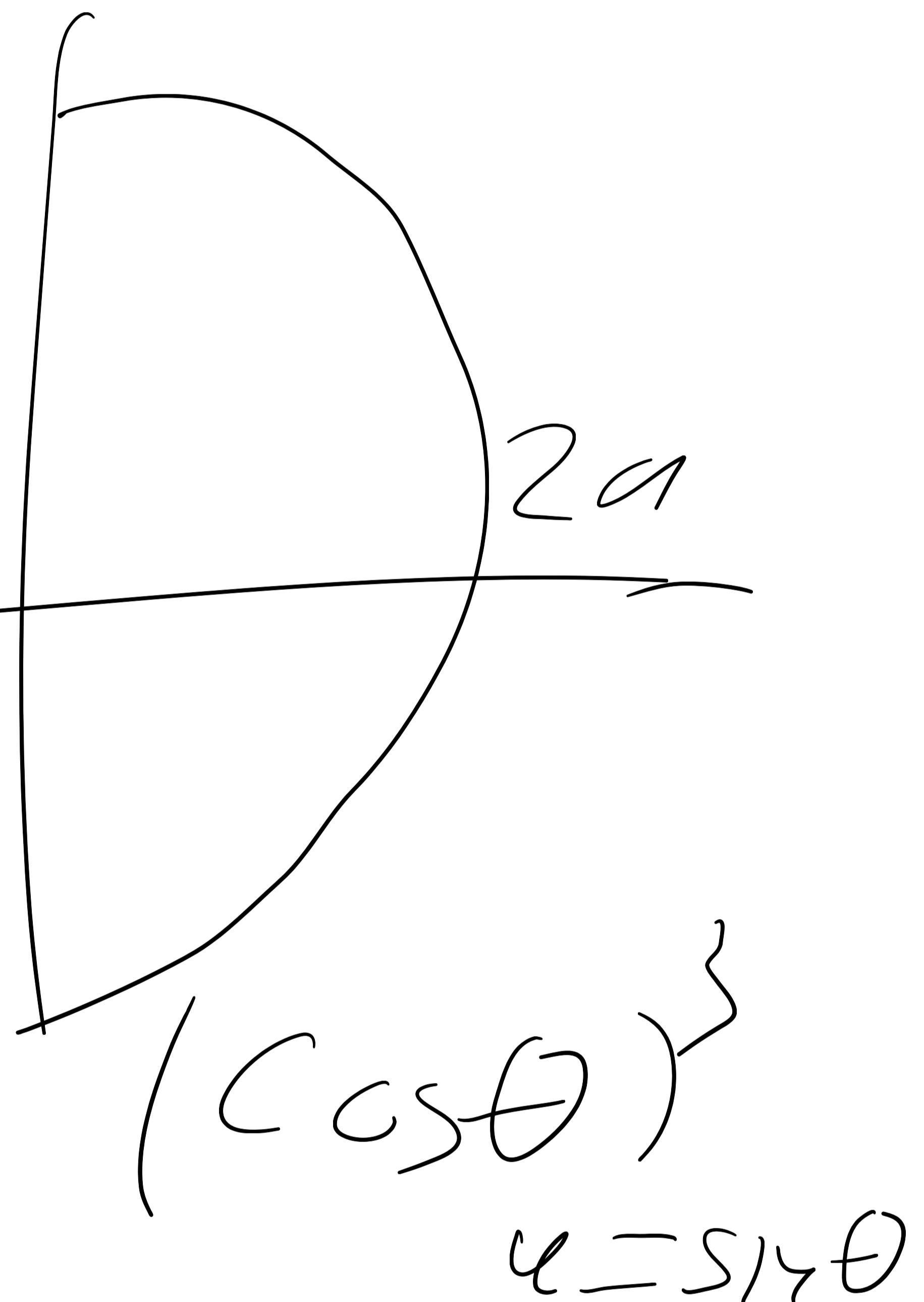
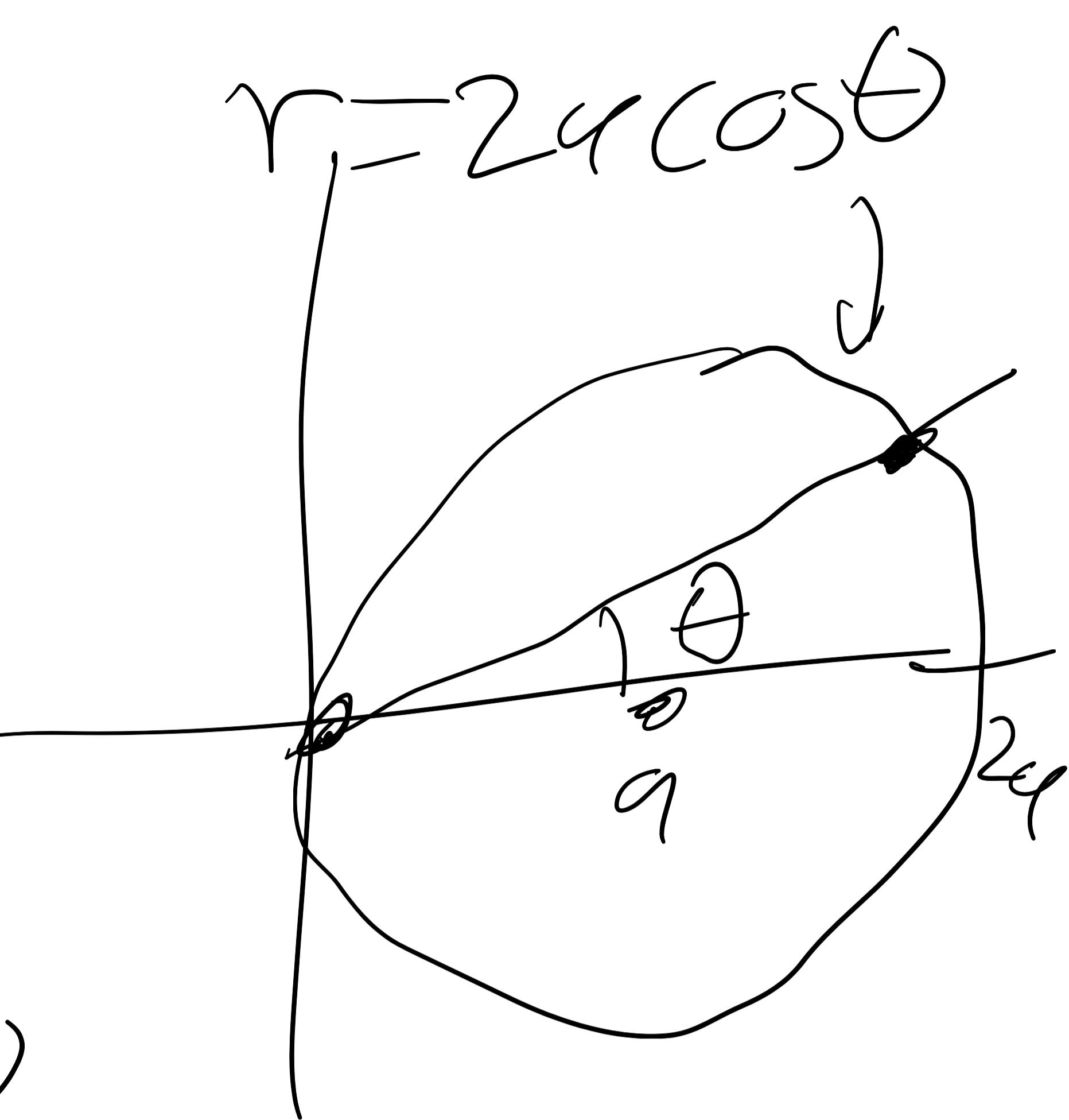
$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2a \cos \theta} r \cdot r dr d\theta$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2a \cos \theta} (2a \cos \theta)^3 d\theta$$

$$= \frac{8a^3}{3} \int_0^{\frac{\pi}{2}} (1 - \sin^2 \theta) (\sin \theta)' d\theta$$

$$= \frac{16a^3}{3} \int_0^{\frac{\pi}{2}} (1 - u^2) du =$$

$$= \frac{16a^3}{3} \left(1 - \frac{1}{3}\right) = \frac{32a^3}{9}$$



$$u = \sin \theta$$